Kurs ▮ Cours

3. Algebraische und symbolische Stärken ▮ Forces algébriques et symboliques

Die Gliederung dieses Kurses folgt in groben Zügen dem Buch von Nancy Blachman: A Practical Approach....
Hinweis: Kapitel 3 lesen!
Run mit WIN+*Mathematica* Version 5.2

▮ L'articulation de ce cours correspond à peu près à celle du livre de Nancy Blachman: A Practical Approach....
Indication: Lire le chapitre 3.
Testé avec *Mathematica* version 5.2+WIN

3.1. Algebra ▮ Algèbre

Mathematica kann algebraische Ausdrücke manipulieren.
▮ *Mathematica* peut manipuler les expressions algébriques

3.1.1. Studiere: ▮ Etudie:

In[1]:= **?Apart**

```
Apart[expr] rewrites a rational expression as a sum of terms with minimal denominators.
   Apart[expr, var] treats all variables other than var as constants. Mehr…
```

In[2]:= **?Cancel**

```
Cancel[expr] cancels out common factors in the numerator and denominator of expr. Mehr…
```

In[3]:= **?Collect**

```
Collect[expr, x] collects together terms involving the same powers of objects
   matching x. Collect[expr, {x1, x2, ... }] collects together terms that involve
   the same powers of objects matching x1, x2, ... . Collect[expr, var, h] applies
   h to the expression that forms the coefficient of each term obtained. Mehr…
```

In[4]:= **?Expand**

Expand[expr] expands out products and positive integer powers in expr. Expand[expr, patt] leaves unexpanded any parts of expr that are free of the pattern patt. Mehr...

In[5]:= **?ExpandAll**

ExpandAll[expr] expands out all products and integer powers in any part of expr. ExpandAll[expr, patt] avoids expanding parts of expr that do not contain terms matching the pattern patt. Mehr...

In[6]:= **?ExpandDenominator**

ExpandDenominator[expr] expands out products and powers that appear as denominators in expr. Mehr...

In[7]:= **?ExpandNumerator**

ExpandNumerator[expr] expands out products and powers that appear in the numerator of expr. Mehr...

In[8]:= **?Factor**

Factor[poly] factors a polynomial over the integers. Factor[poly, Modulus->p] factors a polynomial modulo a prime p. Factor[poly, Extension->{a1, a2, ... }] factors a polynomial allowing coefficients that are rational combinations of the algebraic numbers ai. Mehr...

In[9]:= **?Simplify**

Simplify[expr] performs a sequence of algebraic and other transformations on expr, and returns the simplest form it finds. Simplify[expr, assum] does simplification using assumptions. Mehr...

In[10]:= **?Short**

Short[expr] prints as a short form of expr, less than about one line long. Short[expr, n] prints as a form of expr about n lines long. Mehr...

In[11]:= **?Together**

Together[expr] puts terms in a sum over a common denominator, and cancels factors in the result. Mehr...

In[12]:=
```
(* Needs["Algebra`Trigonometry`"] *)
(* Old version *)
```

In[14]:= **?TrigExpand**

TrigExpand[expr] expands out trigonometric functions in expr. Mehr...

In[15]:= **?TrigFactor**

TrigFactor[expr] factors trigonometric functions in expr. Mehr...

In[16]:= **?TrigReduce**

TrigReduce[expr] rewrites products and powers of trigonometric functions in expr in terms of trigonometric functions with combined arguments. Mehr...

In[17]:=
```
(* ?ComplexToTrig *)
(* Old version *)
```

```
In[19]:= (* ?*Trig* *)
         (* Old version *)

In[21]:= (* ?TrigToComplex *)
         (* Old version *)

In[23]:= ?*Complex*
```

System`

```
Complex    ComplexExpand    ComplexityFunction
Complexes  ComplexInfinity
```

```
In[24]:= ?ExpToTrig
```

ExpToTrig[expr] converts exponentials in expr to trigonometric functions. Mehr...

```
In[25]:= ?TrigToExp
```

TrigToExp[expr] converts trigonometric functions in expr to exponentials. Mehr...

```
In[26]:= ?Sum
```

Sum[f, {i, imax}] evaluates the sum of the expressions f as evaluated for each i from 1 to imax. Sum[f, {i, imin, imax}] starts with i = imin. Sum[f, {i, imin, imax, di}] uses steps di. Sum[f, {i, imin, imax}, {j, jmin, jmax}, ...] evaluates a sum over multiple indices. Mehr...

```
In[27]:= Sum[i^2,{i, n}]
```

Out[27]= $\frac{1}{6} n (1 + n) (1 + 2 n)$

```
In[28]:= (* Needs["Algebra`GosperSum`"] *)
         (* Old: *)

In[30]:= (* ?GosperSum *)
         (* Old: *)

In[32]:= ?*Sum*
```

System`

```
NSum            NSumTerms  Sum
NSumExtraTerms  RootSum
```

Was in alten Versionen einmal existiert hat, braucht in neuen nicht mehr zu existieren...

■ Ce qui existait dans de vieilles versions, n'existe pas forcement dans des versions nouvelles.

```
In[33]:= $Version (*Mathematica-Version*)

Out[33]= 5.2 for Microsoft Windows (June 20, 2005)

In[34]:= ?$Version
```

$Version is a string that represents the version of Mathematica you are running. Mehr...

```
In[35]:= (* Needs["Algebra`SymbolicSum`"] *)
         (* Old: *)
```

```
In[37]:= Sum[i^2,{i, n}]
```

Out[37]= $\frac{1}{6}\, n\,(1+n)\,(1+2\,n)$

```
In[38]:= ?Limit
```

```
Limit[expr, x->x0] finds the limiting value of expr when x approaches x0. Mehr…
```

```
In[39]:= ?Integrate
```

```
Integrate[f, x] gives the indefinite integral of f with respect to x.
   Integrate[f, {x, xmin, xmax}] gives the definite integral of f with respect
   to x from xmin to xmax. Integrate[f, {x, xmin, xmax}, {y, ymin, ymax}]
   gives a multiple definite integral of f with respect to x and y. Mehr…
```

```
In[40]:= ?Series
```

```
Series[f, {x, x0, n}] generates a power series expansion for f
   about the point x = x0 to order (x - x0)^n. Series[f, {x, x0, nx}, {y, y0,
   ny}] successively finds series expansions with respect to y, then x. Mehr…
```

```
In[41]:= ?DSolve
```

```
DSolve[eqn, y, x] solves a differential equation for the function y, with independent
   variable x. DSolve[{eqn1, eqn2, ...  }, {y1, y2, ...  }, x] solves a list of differential
   equations. DSolve[eqn, y, {x1, x2, ... }] solves a partial differential equation. Mehr…
```

```
In[42]:= ??DSolve
```

```
DSolve[eqn, y, x] solves a differential equation for the function y, with independent
   variable x. DSolve[{eqn1, eqn2, ...  }, {y1, y2, ...  }, x] solves a list of differential
   equations. DSolve[eqn, y, {x1, x2, ... }] solves a partial differential equation. Mehr…

Attributes[DSolve] = {Protected}

Options[DSolve] = {GeneratedParameters → C}
```

3.1.2. Probiere: ▮ Essaie:

Ueberlege Dir, was durch die folgenden Operationen bewirkt wird!
▮ Réfléchis à ce qui se produit par les opérations suivantes!

```
In[43]:= Expand[(x + 2y + z)^4]
```

Out[43]= $x^4+8\,x^3\,y+24\,x^2\,y^2+32\,x\,y^3+16\,y^4+4\,x^3\,z+24\,x^2\,y\,z+48\,x\,y^2\,z+32\,y^3\,z+6\,x^2\,z^2+24\,x\,y\,z^2+24\,y^2\,z^2+4\,x\,z^3+8\,y\,z^3+z^4$

```
In[44]:= Factor[%]
```

Out[44]= $(x+2\,y+z)^4$

```
In[45]:= Factor[6x^3 + 35 x^2 y + 58 x y^2 + 21 y^3]
```

Out[45]= $(2\,x+y)\,(x+3\,y)\,(3\,x+7\,y)$

3.2. Gleichungen lösen
■ Résoudre des équations

3.2.1. Algebraisch exakte Lösung:
■ Solution exacte algébriquement:

NRoots rechnet eine Näherung. "||" bedeutet dabei das logische "oder", "or".
■ NRoots calcule une approximation. "||" signifie le "ou" logique.

```
In[46]:= NRoots[x^3 - 3 x^2 - 17 x + 51 == 0, x]

Out[46]= x == -4.12311 || x == 3. || x == 4.12311
```

"||" ist das logische "oder".
■ "||" est le "ou" logique.

```
In[47]:= ??||

e1 || e2 || ...  is the logical OR function. It evaluates its arguments in order, giving
   True immediately if any of them are True, and False if they are all False. Mehr...

Attributes[Or] = {Flat, HoldAll, OneIdentity, Protected}
```

Gewisse algebraische Gleichungen oder Gleichungssysteme kann man jedoch exakt lösen. Dies geschieht mit "Solve".
■ On peut résoudre exactement certaines équations algébriques ou certains systèmes d'équations. Cela avec "Solve".

```
In[48]:= Solve[x^3 - 3 x^2 - 17 x + 51 == 0, x]
```

Out[48]= $\{\{x \to 3\}, \{x \to -\sqrt{17}\}, \{x \to \sqrt{17}\}\}$

Angenähert: ■ Approximé

```
In[49]:= % // N

Out[49]= {{x → 3.}, {x → -4.12311}, {x → 4.12311}}
```

Uebersichtlicher dargestellt:
■ Représenté d'une façon plus claire:

```
In[50]:= ColumnForm[Solve[{  x y + 5x + 6y == 7,
                             x^2 + 5x + 7y == 8}]]
```

Out[50]= $\{y \to \frac{2}{7}, x \to 1\}$
$\{y \to -5 - \sqrt{37}, x \to -6 - \sqrt{37}\}$
$\{y \to -5 + \sqrt{37}, x \to -6 + \sqrt{37}\}$

Der Output besteht aus einer Menge von Ersetzungsregeln.
Wie lässt sich damit weiterrechnen?
■ L'output est composé d'une quantité de règles de remplacement. Comment continue-t-on de calculer avec cela?

3.2.2. Weiterverwendung der Lösung: ■ Réutilisation de la solution:

"/." bedeutet "ersetze jedes" resp. "ReplaceAll".
■ "/." signifie "remplace tout" resp. "Remplace All".

```
In[51]:= ?/.

        expr /. rules applies a rule or list of rules in
           an attempt to transform each subpart of an expression expr. Mehr…

In[52]:= ?ReplaceAll

        expr /. rules applies a rule or list of rules in
           an attempt to transform each subpart of an expression expr. Mehr…

In[53]:= ?Replace

        Replace[expr, rules] applies a rule or list of rules in an
           attempt to transform the entire expression expr. Replace[expr, rules,
           levelspec] applies rules to parts of expr specified by levelspec. Mehr…
```

Studiere die folgenden Anwendungen:
■ Etudie les applications suivantes:

```
In[54]:= a = x /. x -> 7

Out[54]= 7

In[55]:= a

Out[55]= 7

In[56]:= x

Out[56]= x
```

Weiter: ■ Ensuite:

```
In[57]:= x + 5y /. {x -> 1, y -> 2/7}
```

Out[57]= $\frac{17}{7}$

```
In[58]:= x + 5y

Out[58]= x + 5 y
```

Weiter: ■ Encore:

```
In[59]:= Solve[2 x^5 + 12 x^2 + 18x - 14 == 0]
```

Out[59]= $\{\{x \to \text{Root}[-7 + 9\,\#1 + 6\,\#1^2 + \#1^5\,\&, 1]\},$
$\{x \to \text{Root}[-7 + 9\,\#1 + 6\,\#1^2 + \#1^5\,\&, 2]\}, \{x \to \text{Root}[-7 + 9\,\#1 + 6\,\#1^2 + \#1^5\,\&, 3]\},$
$\{x \to \text{Root}[-7 + 9\,\#1 + 6\,\#1^2 + \#1^5\,\&, 4]\}, \{x \to \text{Root}[-7 + 9\,\#1 + 6\,\#1^2 + \#1^5\,\&, 5]\}\}$

Die Lösung hier ist nur symbolisch. Eine allgemeine algebraisch exakte Lösungsformel
für Gleichungen mit Polynomen 5. Grades existiert nicht. Approximativ allerdings geht es:

■ La solution n'est que symbolique. Une forme de solution algébrique générale exacte pour les équations avec polynômes du 5e degré n'existe pas. Mais une approximation est possible (existe):

```
In[60]:= N[%]

Out[60]= {{x → 0.561438}, {x → -1.46299 - 0.832383 i},
         {x → -1.46299 + 0.832383 i}, {x → 1.18228 - 1.73288 i}, {x → 1.18228 + 1.73288 i}}
```

3.3. Vereinfachungen
■ Simplifications

3.3.1. Allgemeines
■ Généralités

Mathematica kann nicht die Frage beantworten, welche Outputform eines Ausdrucks für den Verwender der einfachste ist. Man muss den Ausdruck selbst in die gewünschte Form bringen. Dafür gibt es Hilfsmittel wie z.B. "Simplify":
■ *Mathematica* ne sait pas répondre à la question, laquelle des formes de l'Output d'une expression est la plus simple pour l'utilisateur. Il faut donner soi-même la forme souhaitée à l'expression. Pour cela il y a des moyens tels que p.ex. "Simplyfy".

```
In[61]:= Solve[(a d x^3 + 14 b d x^2 + 14^2 c d x)
                     == 0, x] (14b + 14a)/(-c + b)
```

Out[61]= $\left\{\left\{\frac{(98+14\,b)\,(x\to 0)}{b-c}\right\}, \left\{\frac{(98+14\,b)\left(x\to -b-\sqrt{b^2-28\,c}\right)}{b-c}\right\}, \left\{\frac{(98+14\,b)\left(x\to -b+\sqrt{b^2-28\,c}\right)}{b-c}\right\}\right\}$

```
In[62]:= Simplify[%]
```

Out[62]= $\left\{\left\{\frac{14\,(7+b)\,(x\to 0)}{b-c}\right\}, \left\{\frac{14\,(7+b)\left(x\to -b-\sqrt{b^2-28\,c}\right)}{b-c}\right\}, \left\{\frac{14\,(7+b)\left(x\to -b+\sqrt{b^2-28\,c}\right)}{b-c}\right\}\right\}$

```
In[63]:= ??Simplify

    Simplify[expr] performs a sequence of algebraic and
       other transformations on expr, and returns the simplest form it finds.
       Simplify[expr, assum] does simplification using assumptions. Mehr...

    Attributes[Simplify] = {Protected}

    Options[Simplify] = {Assumptions :→ $Assumptions, ComplexityFunction → Automatic,
      TimeConstraint → 300, TransformationFunctions → Automatic, Trig → True}

In[64]:= x/((x+2)(x-2))
```

Out[64]= $\frac{x}{(-2+x)\,(2+x)}$

```
In[65]:= Simplify[%]
```

Out[65]= $\frac{x}{-4+x^2}$

```
In[66]:= x/Expand[((x+2)(x-2))]
```

Out[66]= $\frac{x}{-4+x^2}$

```
In[67]:= Expand[x/((x+2)(x-2))]
```

Out[67]= $\frac{x}{(-2+x)\ (2+x)}$

```
In[68]:= x/Simplify[((x+2)(x-2))]
```

Out[68]= $\frac{x}{-4+x^2}$

3.3.2. Andere Vorgehensweisen:
▮ Autres façons de procéder

Beispiel: Partialbruchzerlegung mit "Apart". Probiere aus.:
▮ Exemple: Décomposition de fractions partielles en utilisant "Apart":

```
In[69]:= x/((x+2)(x-2))
```

Out[69]= $\frac{x}{(-2+x)\ (2+x)}$

```
In[70]:= Apart[%]
```

Out[70]= $\frac{1}{2\ (-2+x)}+\frac{1}{2\ (2+x)}$

Retour: ▮ Retour:

```
In[71]:= Together[%]
```

Out[71]= $\frac{x}{(-2+x)\ (2+x)}$

Man beachte die Reihenfolge der Terme im Output. Zuerst Konstanten, dann Symbole, alphabetisch, Terme nach steigendem Grad etc.:
▮ Il faut tenir compte des termes dans le output. D'abort les constantes, puis les symboles, en ordre alphabétique, termes selon le dégré en augmentant etc.:

```
In[72]:= Expand[(1 + 2y + 3z)^3]
```

Out[72]= $1+6\ y+12\ y^2+8\ y^3+9\ z+36\ y\ z+36\ y^2\ z+27\ z^2+54\ y\ z^2+27\ z^3$

Umordnen: ▮ Réordonner:

```
In[73]:= Collect[%, y]
```

Out[73]= $1+8\ y^3+9\ z+27\ z^2+27\ z^3+y^2\ (12+36\ z)+y\ (6+36\ z+54\ z^2)$

3.3.3. Trigonometrische Ausdrücke:
▮ Expressions trigonométriques:

In[74]:= **Expand[Sin[x]/Cos[x]]**

Out[74]= Tan[x]

In[75]:= **Expand[(Sin[x]+Cos[x])/Sin[x] (Sin[x]^2 + Cos[x]^2), Trig -> False]**

Out[75]= $\text{Cos}[x]^2 + \text{Cos}[x]^2\,\text{Cot}[x] + \text{Cos}[x]\,\text{Sin}[x] + \text{Sin}[x]^2$

In[76]:= **?Trig**

```
Trig is an option for algebraic manipulation functions which specifies whether
  trigonometric functions should be treated as rational functions of exponentials.
```

In[77]:= **Expand[(Sin[x]+Cos[x])/Sin[x] (Sin[x]^2 + Cos[x]^2), Trig -> True]**

Out[77]= 1 + Cot[x]

In[78]:= **(* Old *)**
TrigToComplex[Sin[x]]

Out[79]= TrigToComplex[Sin[x]]

In[80]:= **? TrigToComplex**

```
Global`TrigToComplex
```

In[81]:= **TrigToExp[Sin[x]]**

Out[81]= $\frac{1}{2}\,i\,e^{-i x} - \frac{1}{2}\,i\,e^{i x}$

In[82]:= **(* Needs["Algebra`Trigonometry`"] *)**
(* Old *)

In[84]:= **? *Trig***

System`

```
ExpToTrig  TrigExpand  TrigFactorList  TrigToExp
Trig       TrigFactor  TrigReduce
```

Global`

```
TrigToComplex
```

3.3.4. Lange Resultate:
▮ Résultats longs

Mathematica bietet die Möglichkeit, Resultate abzukürzen:

▮ *Mathematica* offre la possibilité d'abréger les résultats:

In[85]:= **Short[Expand[(x + 2y + 3z)^9]]**

Out[85]//Short=

$x^9 + 18 x^8 y + \ll 51 \gg + 118098 y z^8 + 19683 z^9$

In[86]:= **?Short**

```
Short[expr] prints as a short form of expr, less than about one line
   long. Short[expr, n] prints as a form of expr about n lines long. Mehr…
```

Mit der globalen Variablen "$PrePrint" lässt sich das Abkürzen der Ausgabe generalisieren:

■ Avec la variable globale "$PrePrint" on peut généralement abréger la sortie:

In[87]:= **$PrePrint = Short**

Out[87]= Short

In[88]:= **Expand[(x + 2y + 3z)^9]**

Out[88]= $x^9 + 18 x^8 y + \ll 51 \gg + 118098 y z^8 + 19683 z^9$

In[89]:= **Expand[(x + 2y + 3z)^11]**

Out[89]= $x^{11} + 22 x^{10} y + \ll 74 \gg + 1299078 y z^{10} + 177147 z^{11}$

In[90]:= **Expand[(x + 2y + 3z)^5]**

Out[90]= $x^5 + 10 x^4 y + 40 x^3 y^2 + \ll 16 \gg + 810 y z^4 + 243 z^5$

Trotzdem kann man das ganze Resultat abrufen mit "Print":

■ En tout cas on peut appeler le résultat entier avec "Print":

In[91]:= **Print[%]**

$x^5 + 10 x^4 y + 40 x^3 y^2 + 80 x^2 y^3 + 80 x y^4 + 32 y^5 + 15 x^4 z + 120 x^3 y z + 360 x^2 y^2 z + 480 x y^3 z + 240 y^4 z + 90 x^3 z^2 + 540 x^2 y z^2 + 1080 x y^2 z^2 + 720 y^3 z^2 + 270 x^2 z^3 + 1080 x y z^3 + 1080 y^2 z^3 + 405 x z^4 + 810 y z^4 + 243 z^5$

Alter Zustand: ■ Ancien état:

In[92]:= **$PrePrint = .**

In[93]:= **Expand[(x + 2y + 3z)^9]**

Out[93]= $x^9 + 18 x^8 y + 144 x^7 y^2 + 672 x^6 y^3 + 2016 x^5 y^4 + 4032 x^4 y^5 + 5376 x^3 y^6 + 4608 x^2 y^7 + 2304 x y^8 + 512 y^9 + 27 x^8 z + 432 x^7 y z + 3024 x^6 y^2 z + 12096 x^5 y^3 z + 30240 x^4 y^4 z + 48384 x^3 y^5 z + 48384 x^2 y^6 z + 27648 x y^7 z + 6912 y^8 z + 324 x^7 z^2 + 4536 x^6 y z^2 + 27216 x^5 y^2 z^2 + 90720 x^4 y^3 z^2 + 181440 x^3 y^4 z^2 + 217728 x^2 y^5 z^2 + 145152 x y^6 z^2 + 41472 y^7 z^2 + 2268 x^6 z^3 + 27216 x^5 y z^3 + 136080 x^4 y^2 z^3 + 362880 x^3 y^3 z^3 + 544320 x^2 y^4 z^3 + 435456 x y^5 z^3 + 145152 y^6 z^3 + 10206 x^5 z^4 + 102060 x^4 y z^4 + 408240 x^3 y^2 z^4 + 816480 x^2 y^3 z^4 + 816480 x y^4 z^4 + 326592 y^5 z^4 + 30618 x^4 z^5 + 244944 x^3 y z^5 + 734832 x^2 y^2 z^5 + 979776 x y^3 z^5 + 489888 y^4 z^5 + 61236 x^3 z^6 + 367416 x^2 y z^6 + 734832 x y^2 z^6 + 489888 y^3 z^6 + 78732 x^2 z^7 + 314928 x y z^7 + 314928 y^2 z^7 + 59049 x z^8 + 118098 y z^8 + 19683 z^9$

3.4. Summation
▮ Sommation

Mathematica bietet die Möglichkeit, z.B. Summen mit variablem oberen Summationsindex zu berechnen:

▮ *Mathematica* offre la possibilité p.ex. de calculer des sommes avec un index de sommation supérieur variable:

```
In[94]:= ?*Sum*
```

System`

```
NSum            NSumTerms  Sum
NSumExtraTerms  RootSum
```

```
In[95]:= ?Sum
```

```
Sum[f, {i, imax}] evaluates the sum of the expressions f as
   evaluated for each i from 1 to imax. Sum[f, {i, imin, imax}] starts with
   i = imin. Sum[f, {i, imin, imax, di}] uses steps di. Sum[f, {i, imin,
   imax}, {j, jmin, jmax}, ... ] evaluates a sum over multiple indices. Mehr…
```

```
In[96]:= (* Needs["Algebra`SymbolicSum`"] *)
         (* Old *)
```

```
In[98]:= Sum[i^3,{i, n}]
```

Out[98]= $\frac{1}{4} n^2 (1 + n)^2$

Probiere eigene Aufgaben aus:

▮ Essaie des problèmes que tu inventes toi-même:

```
In[99]:=
```

3.5. Calculus (Differential- und Integralrechnung)
▮ Calculus (Calcul différentiel et intégral)

3.5.1. Unbestimmte Integrale
▮ Intégrals indéterminées

Beispiele: ▮ Exemple:

```
In[99]:= Integrate[Cos[x], x]

Out[99]= Sin[x]
```

In[100]:=
```
Integrate[x^4 Cos[x], x]
```
Out[100]=

$$4\,x\,(-6+x^2)\,\mathrm{Cos}[x]+(24-12\,x^2+x^4)\,\mathrm{Sin}[x]$$

In[101]:=
```
1/(2 + 3 x^2)^3
```
Out[101]=

$$\frac{1}{(2+3\,x^2)^3}$$

In[102]:=
```
Integrate[1/(2 + 3 x^2)^3, x]
```
Out[102]=

$$\frac{x}{8\,(2+3\,x^2)^2}+\frac{3\,x}{32\,(2+3\,x^2)}+\frac{1}{32}\sqrt{\frac{3}{2}}\,\mathrm{ArcTan}\left[\sqrt{\frac{3}{2}}\,x\right]$$

Kontrolle: Differenzieren!
■ Contrôle: Différencier!

In[103]:=
```
D[%, x]
```
Out[103]=

$$\frac{3}{64\,\left(1+\frac{3\,x^2}{2}\right)}-\frac{3\,x^2}{2\,(2+3\,x^2)^3}+\frac{1}{8\,(2+3\,x^2)^2}-\frac{9\,x^2}{16\,(2+3\,x^2)^2}+\frac{3}{32\,(2+3\,x^2)}$$

Das Resultat kann durchaus anders aussehen. Es muss vielleicht umgeformt werden.
■ Le résultat peut être différent. Il faut peut-être le transformer.

In[104]:=
```
Simplify[%]
```
Out[104]=

$$\frac{1}{(2+3\,x^2)^3}$$

3.5.2. Bestimmte Integrale
■ Intégrales déterminées

Mathematica kann bestimmte Integrale exakt rechnen. Probiere aus:
■ *Mathematica* sait calculer exactement des intégrales déterminées. Essaie:

In[105]:=
```
?Integrate
```
```
Integrate[f, x] gives the indefinite integral of f with respect to x.
   Integrate[f, {x, xmin, xmax}] gives the definite integral of f with respect
   to x from xmin to xmax. Integrate[f, {x, xmin, xmax}, {y, ymin, ymax}]
   gives a multiple definite integral of f with respect to x and y. Mehr…
```

In[106]:=

```
?*Integ*
```

System`

```
CoshIntegral      Integer          Integrate
CosIntegral       IntegerDigits    LogIntegral
ExpIntegralE      IntegerExponent  NIntegrate
ExpIntegralEi     IntegerPart      SinhIntegral
FactorInteger     IntegerQ         SinIntegral
GaussianIntegers  Integers
```

In[107]:=

```
Integrate[Exp[x],{x, -1, 1}]
```

Out[107]=

$$-\frac{1}{e}+e$$

In[108]:=

```
N[%]
```

Out[108]=

2.3504

Numerisch geht es natürlich auch. Probiere aus:

- Il va de soi qu'on peut le faire numériquement. Essaie:

In[109]:=

```
NIntegrate[Exp[x],{x, -1, 1}]
```

Out[109]=

2.3504

Zu gewissen Funktionen kann keine einfache Stammfunktion gefunden werden:

- Pour certaines fonctions on ne trouve pas de fonction primitive (intégrale) simple:

In[110]:=

```
Integrate[Sin[Sin[x]],{x, 0, 1}]
```

Out[110]=

$$\int_0^1 \mathrm{Sin}[\mathrm{Sin}[x]]\, dx$$

Numerisch geht es natürlich. Probiere aus:

- Numériquement ça va. Essaie:

In[111]:=

```
NIntegrate[Sin[Sin[x]],{x, 0, 1}]
```

Out[111]=

0.430606

3.5.3. Integrale von Funktionen mit Polen
▮ Intégrales de fonctions avec pôles

Bei Funktionen mit Polen können Fehler auftreten, wenn man einfach eine Stammfunktion bestimmt und die Grenzen einsetzt. Prüfe:

▮ Lors de fonctions avec pôles, des erreurs peuvent appataître, si on détermine simplement une fonction primitive (intégrale) et fixe les limites. Examine:

In[112]:=

```
Integrate[1/(x-Pi/100)^2,{x, -1, 1}]
```

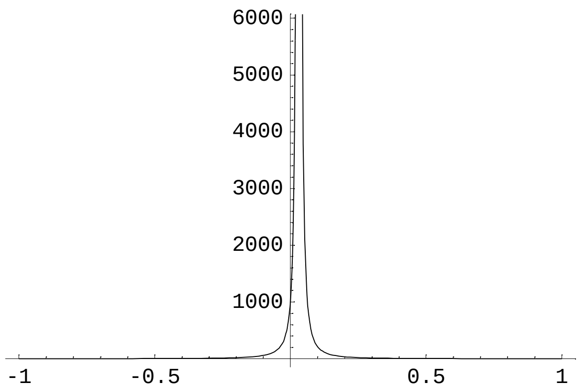
```
Integrate::idiv :
 Integral of FractionBox["1", SuperscriptBox[RowBox[{"(", RowBox[{RowBox[{"-", FractionBox["π", RowBox[
   {"", \(<< 3 >>\), ""}]]}], "+", "x"}], ")"}], "2"]] does not converge on {-1, 1}. Mehr…
```

Out[112]=

$$\int_{-1}^{1} \frac{1}{\left(-\frac{\pi}{100}+x\right)^2}\,dx$$

In[113]:=

```
Plot[1/(x-Pi/100)^2,{x, -1, 1}];
```



In[114]:=

```
NIntegrate[1/(x-Pi/100)^2,{x, -1, 1}]
```

```
NIntegrate::ncvb : NIntegrate failed to converge to
   prescribed accuracy after 7 recursive bisections in x near x = 0.0390625`. Mehr…
```

Out[114]=

200852.

In[115]:=

```
Integrate[1/x^2,{x, -1, 1}]
```

Integrate::idiv : Integral of $\frac{1}{x^2}$ does not converge on {-1, 1}. Mehr…

Out[115]=

$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

In[116]:=

```
NIntegrate[1/x^2,{x, -1, 1}]
```

```
NIntegrate::slwcon :
 Numerical integration converging too slowly; suspect one of the following: singularity, value of
   the integration being 0, oscillatory integrand, or insufficient WorkingPrecision. If your
   integrand is oscillatory try using the option Method->Oscillatory in NIntegrate. Mehr…

NIntegrate::ncvb : NIntegrate failed to converge to prescribed
   accuracy after 7 recursive bisections in x near x = 4.369993747903698`*^-57. Mehr…
```

Out[116]=

$2.481310356720057 \times 10^{3498}$

In[117]:=

```
??NIntegrate
```

```
NIntegrate[f, {x, xmin, xmax}] gives a numerical approximation
   to the integral of f with respect to x from xmin to xmax. Mehr…

Attributes[NIntegrate] = {HoldAll, Protected}

Options[NIntegrate] =
 {AccuracyGoal → ∞, Compiled → True, EvaluationMonitor → None, GaussPoints → Automatic,
  MaxPoints → Automatic, MaxRecursion → 6, Method → Automatic, MinRecursion → 0,
  PrecisionGoal → Automatic, SingularityDepth → 4, WorkingPrecision → MachinePrecision}
```

Es kann z.B. angegeben werden, dass bei 0 eine Singularität (Polstelle) ist. Prüfe:

■ On peut indiquer p.ex. qu'il y a une singularité (pôle) près de 0. Examine:

In[118]:=

```
NIntegrate[1/x^2,{x, -1, 0, 1}]
```

```
NIntegrate::slwcon :
 Numerical integration converging too slowly; suspect one of the following: singularity, value of
   the integration being 0, oscillatory integrand, or insufficient WorkingPrecision. If your
   integrand is oscillatory try using the option Method->Oscillatory in NIntegrate. Mehr…

NIntegrate::ncvb : NIntegrate failed to converge to prescribed
   accuracy after 7 recursive bisections in x near x = 4.369993747903698`*^-57. Mehr…
```

Out[118]=

$2.481310356720057 \times 10^{3498}$

3.5.4. Mehrfachintegrale
■ Intégrales multiples

f(x, y) kann über x und y integriert werden. Probiere:

■ f(x, y) peut être intégré sur x et y. Essaie:

In[119]:=

```
Integrate[x^2 Sin[y],x, y]
```

Out[119]=

$-\frac{1}{3} x^3 \mathrm{Cos}[y]$

Bei bestimmten Integralen sind die äusseren Grenzen zuerst anzugeben:

■ Pour certains intégrales les limites extérieurs doivent êtres données d'abort.

In[120]:=

```
Integrate[x^2 Sin[y],{x, 0, Pi/3}, {y, 0, x}]
```

Out[120]=

$$\sqrt{3} - \frac{\pi}{3} - \frac{\pi^2}{6\sqrt{3}} + \frac{\pi^3}{81}$$

3.6. Grenzwerte
■ Valeurs limites

Mit "Limits" lassen sich Grenzwerte berechnen. "Unendlich" ist "Infinity":

■ Avec "Limits" on peut calculer des valeurs limites. "Infinie" est "Infinity":

In[121]:=

```
?Limit
```

Limit[expr, x->x0] finds the limiting value of expr when x approaches x0. Mehr…

In[122]:=

```
?Infinity
```

Infinity is a symbol that represents a positive infinite quantity. Mehr…

In[123]:=

```
Limit[Sin[x]/x, x -> 0]
```

Out[123]=

1

In[124]:=

```
Limit[1/x, x -> 0]
```

Out[124]=

∞

Der Wert des Grenzwerts kann davon abhängen, aus welcher Richtung man sich dem problematischen Punkt nähert. Die Richtung kann angegeben werden:

■ La valeur de la valeur limite peut dépendre de la direction de laquelle on s'approche du point problématique.

In[125]:=

```
Limit[1/x, x -> 0, Direction -> 1]
```

Out[125]=

$-\infty$

In[126]:=

```
Limit[1/x, x -> 0, Direction -> -1]
```

Out[126]=

∞

In[127]:=

```
Integrate[E^(-s t) t^n,{t,0,Infinity}]
```

Out[127]=

If[Re[n] > -1 && Re[s] > 0, s^{-1-n} Gamma[1 + n],
 Integrate[$e^{-s\,t}\,t^n$, {t, 0, ∞}, Assumptions → Re[s] ≤ 0 || 1 + Re[n] ≤ 0]]

In[128]:=

```
Integrate[E^(-s t) Sin[w t],{t,0,Infinity}]
```

Out[128]=

$$\text{If}\left[w \in \text{Reals} \,\&\&\, \text{Re}[s] > 0,\ \frac{w}{s^2 + w^2},\ \text{Integrate}[e^{-s\,t}\,\text{Sin}[t\,w],\ \{t, 0, \infty\},\ \text{Assumptions} \to \text{Re}[s] \le 0\ ||\ w \notin \text{Reals}]\right]$$

3.7. Potenzreihen
▮ Séries de puissances

Beispiele von Potenzreihenentwicklungen:

▮ Exemples de développements de séries de puissances:

In[129]:=

```
Series[f[x],{x,0,9}]
```

Out[129]=

$$f[0] + f'[0]\,x + \frac{1}{2} f''[0]\,x^2 + \frac{1}{6} f^{(3)}[0]\,x^3 + \frac{1}{24} f^{(4)}[0]\,x^4 + \frac{1}{120} f^{(5)}[0]\,x^5 + \frac{1}{720} f^{(6)}[0]\,x^6 + \frac{f^{(7)}[0]\,x^7}{5040} + \frac{f^{(8)}[0]\,x^8}{40320} + \frac{f^{(9)}[0]\,x^9}{362880} + O[x]^{10}$$

In[130]:=

```
Series[f[x],{x,1,6}]
```

Out[130]=

$$f[1] + f'[1]\,(x-1) + \frac{1}{2} f''[1]\,(x-1)^2 + \frac{1}{6} f^{(3)}[1]\,(x-1)^3 + \frac{1}{24} f^{(4)}[1]\,(x-1)^4 + \frac{1}{120} f^{(5)}[1]\,(x-1)^5 + \frac{1}{720} f^{(6)}[1]\,(x-1)^6 + O[x-1]^7$$

In[131]:=

```
Series[E^x,{x,0,9}]
```

Out[131]=

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + O[x]^{10}$$

In[132]:=

```
Series[Cos[x],{x,0,9}]
```

Out[132]=

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} + O[x]^{10}$$

Solch ein Output ist noch kein algebraischer Ausdruck, mit dem weitergerechnet werden
kann, denn die angegebene Ordnung der nachfolgenden Glieder ist kein Term. Mit "Normal" kann diese Angabe entfernt werden. Probiere:

Un tel Output n'est pas encore une expression algébrique, avec laquelle on peut continuer de calculer, car l'ordre donné des membres suivants n'est pas un terme. On peut effacer ce problème par "Normal". Essaie:

In[133]:=

```
a = Normal[Series[E^x,{x,0,9}]]
```

Out[133]=

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880}$$

In[134]:=
```
b = Normal[Series[Cos[x],{x,0,9}]]
```

Out[134]=
$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}$$

In[135]:=
```
b = Normal[Series[Cos[x],{x,0,9}]]
```

Out[135]=
$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}$$

In[136]:=
```
b /. x->1
```

Out[136]=
$$\frac{4357}{8064}$$

In[137]:=
```
a + b
```

Out[137]=
$$2+x+\frac{x^3}{6}+\frac{x^4}{12}+\frac{x^5}{120}+\frac{x^7}{5040}+\frac{x^8}{20160}+\frac{x^9}{362880}$$

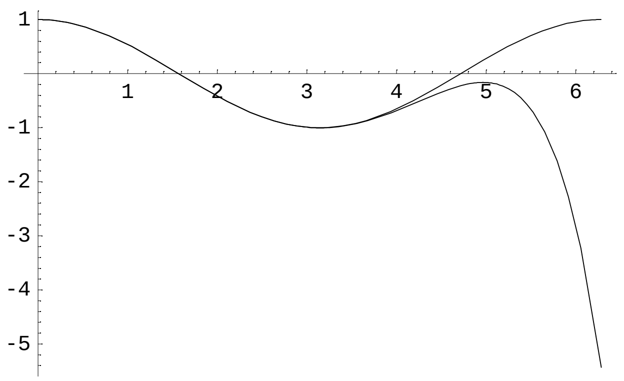
Etwas fürs Auge: ■ Quelque chose pour l'oeil:

In[138]:=
```
Clear[a]
```

In[139]:=
```
prCos[x_]:= Normal[Series[Cos[x],{x,0,11}]];
p = prCos[x]
```

Out[140]=
$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}$$

In[141]:=
```
Plot[{p, Cos[x]}, {x, 0, 2 Pi}];
```



In[142]:=

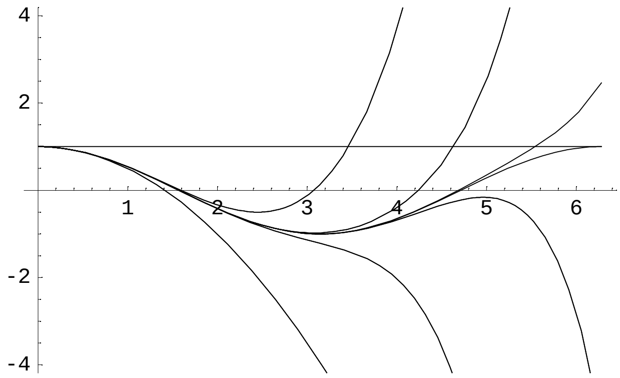
```
a = Append[Evaluate[Table[Normal[Series[Cos[x],{x,0,n}]],{n, 12}]],
Cos[x]]
```

Out[142]=

$$\left\{1,\ 1-\frac{x^2}{2},\ 1-\frac{x^2}{2},\ 1-\frac{x^2}{2}+\frac{x^4}{24},\ 1-\frac{x^2}{2}+\frac{x^4}{24},\ 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720},\right.$$
$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720},\ 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320},\ 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320},$$
$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800},\ 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800},$$
$$\left.1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}+\frac{x^{12}}{479001600},\ \mathrm{Cos}[x]\right\}$$

In[143]:=

```
Plot[Evaluate[Table[a[[i]],{i,13}]], {x, 0, 2 Pi}];
```



In[144]:=

```
??Append
```

Append[expr, elem] gives expr with elem appended. Mehr…

Attributes[Append] = {Protected}

3.8. Lösen von Differentialgleichungen
■ Résoudre des équations différentielles

Mathematica kann gewisse einfache Differentialgleichungen exakt lösen. Probiere:

■ *Mathematica* peut résoudre avec exactitude certaines équations différentielles. Essaie:

In[145]:=

```
?DSolve
```

DSolve[eqn, y, x] solves a differential equation for the function y, with independent variable x. DSolve[{eqn1, eqn2, ... }, {y1, y2, ... }, x] solves a list of differential equations. DSolve[eqn, y, {x1, x2, ... }] solves a partial differential equation. Mehr…

In[146]:=

```
DSolve[y''[x] + y'[x] + y[x] == 0, y[x], x]
```

Out[146]=

$$\left\{\left\{y[x]\to e^{-x/2}\,C[2]\,\mathrm{Cos}\left[\frac{\sqrt{3}\,x}{2}\right]+e^{-x/2}\,C[1]\,\mathrm{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right\}\right\}$$

"C[..]" bedeutet eine Konstante. Auch Systeme können behandelt werden. Probiere:

■ "C[..]" signifie une constante. On peut aussi traiter des systèmes. Essaie:

In[147]:=
```
d = DSolve[{y'[x] + y[x] == 0, y[2] == 2}, y[x], x]
```

Out[147]=

$\{\{y[x] \to 2\, e^{2-x}\}\}$

In[148]:=
```
dd =Flatten[d][[1]]
```

Out[148]=

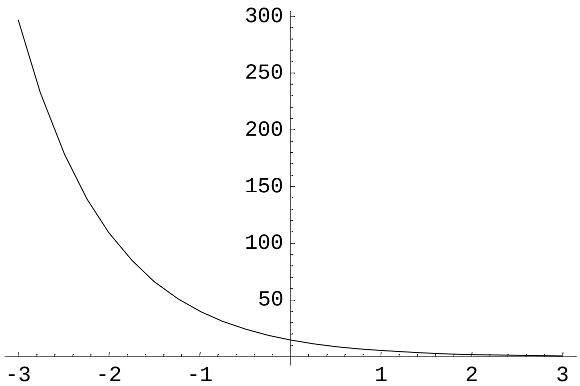
$y[x] \to 2\, e^{2-x}$

In[149]:=
```
ddd = dd[[2]]
```

Out[149]=

$2\, e^{2-x}$

In[150]:=
```
Plot[ddd, {x, -3, 3}];
```



Aufgabe:Behandle das System {r y'[x] + y[x] == 0, y[2] == 2} für
r = -3, -2, -1, 0, 1, 2, 3 und vergleiche die Graphen.

■ Problème: Traite le système {r y'[x] + y[x] == 0, y[2] == 2} pour
r = -3, -2, -1, 0, 1, 2, 3 et compare les graphes.

"Putzmaschine" einsetzen
■ Employer la "machine de nettoyage"

In[151]:=
```
(* Old Form: Remove["Global`@*"] *)
```

In[152]:=
```
Remove["Global`*"]
```