Matrizen und Eigensysteme

```
Remove["Global`*"]
```

Uebung 1

Konstruktion der Matrix

```
eW1=1;eW2=2;;
d={{eW1,0},{0,eW2}};
a={2,-1}; b={3,2};
m={a,b};
mT=Transpose[m];
m1=Inverse[mT];
r=mT.d.m1
```

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}\right\}\right\}$$

```
r//MatrixForm
```

$$\begin{pmatrix} \frac{10}{7} & \frac{6}{7} \\ \frac{2}{7} & \frac{11}{7} \end{pmatrix}$$

```
mT//MatrixForm
```

$$\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

```
MatrixPower[r,1]
```

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}\right\}\right\}$$

```
MatrixPower[r,2]
```

$$\left\{\left\{\frac{16}{7}, \frac{18}{7}\right\}, \left\{\frac{6}{7}, \frac{19}{7}\right\}\right\}$$

```
mT.d.d.m1
```

$$\left\{\left\{\frac{16}{7}, \frac{18}{7}\right\}, \left\{\frac{6}{7}, \frac{19}{7}\right\}\right\}$$

```
d={{eW1,0},{0,eW2}}
```

$$\{\{1, 0\}, \{0, 2\}\}$$

```
e1={1,0}; e2={0,1};
```

Daten

MatrixRank[r]

2

NullSpace[r]

{}

CharacteristicPolynomial[r,x]

$2 - 3\,x + x^2$

CharacteristicPolynomial[d,x]

$2 - 3\,x + x^2$

Tr[r]

3

Tr[d]

3

Det[r]

2

Det[d]

2

RowReduce[r]

{{1, 0}, {0, 1}}

Kern, Im

NullSpace[r]

{}

RowReduce[r]

{{1, 0}, {0, 1}}

RowReduce[r].{x1,x2}

{x1, x2}

Inverse Matrix

rI=Inverse[r]

$$\left\{\left\{\frac{11}{14}, -\frac{3}{7}\right\}, \left\{-\frac{1}{7}, \frac{5}{7}\right\}\right\}$$

rI//MatrixForm

$$\begin{pmatrix} \frac{11}{14} & -\frac{3}{7} \\ -\frac{1}{7} & \frac{5}{7} \end{pmatrix}$$

rI=mT.d.m1

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}\right\}\right\}$$

m1.Inverse[d].mT

$$\left\{\left\{\frac{11}{14}, \frac{3}{7}\right\}, \left\{\frac{1}{7}, \frac{5}{7}\right\}\right\}$$

m1.Inverse[d].mT//MatrixForm

$$\begin{pmatrix} \frac{11}{14} & \frac{3}{7} \\ \frac{1}{7} & \frac{5}{7} \end{pmatrix}$$

EW, EV

eW=Eigenvalues[r]

$$\{2, 1\}$$

eV=Eigenvectors[r]

$$\left\{\left\{\frac{3}{2}, 1\right\}, \{-2, 1\}\right\}$$

Eigenvectors[r]//MatrixForm

$$\begin{pmatrix} \frac{3}{2} & 1 \\ -2 & 1 \end{pmatrix}$$

Eigensystem[r]

$$\left\{\{2, 1\}, \left\{\left\{\frac{3}{2}, 1\right\}, \{-2, 1\}\right\}\right\}$$

Abbildung:

r.{x1,x2}//MatrixForm

$$\begin{pmatrix} \frac{10\,x1}{7} + \frac{6\,x2}{7} \\ \frac{2\,x1}{7} + \frac{11\,x2}{7} \end{pmatrix}$$

r.e1

$$\left\{\frac{10}{7}, \frac{2}{7}\right\}$$

r.e2

$$\left\{\frac{6}{7}, \frac{11}{7}\right\}$$

rI.e1

$$\left\{\frac{10}{7}, \frac{2}{7}\right\}$$

rI.e2

$$\left\{\frac{6}{7}, \frac{11}{7}\right\}$$

{a x1,x2}//MatrixForm

$$\begin{pmatrix} \{2\,x1, -x1\} \\ x2 \end{pmatrix}$$

r.eV[[1]]

```
{3, 2}
```

r.eV[[2]]

```
{-2, 1}
```

rI.eV[[1]]

```
{3, 2}
```

rI.eV[[2]]

```
{-2, 1}
```

r.Transpose[{k1 eV[[1]],k2 eV[[2]]}]

```
{{3 k1, -2 k2}, {2 k1, k2}}
```

rI.Transpose[{k1 eV[[1]],k2 eV[[2]]}]

```
{{3 k1, -2 k2}, {2 k1, k2}}
```

Uebung 2

Konstruktion der Matrix

```
eW1=1;eW2=2; eW3=3;
d={{eW1,0,0},{0,eW2,0},{0,0,eW2}};
a={2,-1,0}; b={3,2,0}; c={0,2,4};
m={a,b,c};
mT=Transpose[m];
m1=Inverse[mT];
r=mT.d.m1
```

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, \frac{3}{14}\right\}, \{0, 0, 2\}\right\}$$

```
r//MatrixForm
```

$$\begin{pmatrix} \frac{10}{7} & \frac{6}{7} & -\frac{3}{7} \\ \frac{2}{7} & \frac{11}{7} & \frac{3}{14} \\ 0 & 0 & 2 \end{pmatrix}$$

```
mT//MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 0 \\ -1 & 2 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

```
MatrixPower[r,1]
```

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, \frac{3}{14}\right\}, \{0, 0, 2\}\right\}$$

```
MatrixPower[r,2]
```

$$\left\{\left\{\frac{16}{7}, \frac{18}{7}, -\frac{9}{7}\right\}, \left\{\frac{6}{7}, \frac{19}{7}, \frac{9}{14}\right\}, \{0, 0, 4\}\right\}$$

```
mT.d.d.m1
```

$$\left\{\left\{\frac{16}{7}, \frac{18}{7}, -\frac{9}{7}\right\}, \left\{\frac{6}{7}, \frac{19}{7}, \frac{9}{14}\right\}, \{0, 0, 4\}\right\}$$

```
e1={1,0,0}; e2={0,1,0}; e3={0,0,1};
```

Daten

```
MatrixRank[r]
```

3

NullSpace[r]

{}

CharacteristicPolynomial[r,x]

$4 - 8\,x + 5\,x^2 - x^3$

CharacteristicPolynomial[d,x]

$4 - 8\,x + 5\,x^2 - x^3$

Tr[r]

5

Tr[d]

5

Det[r]

4

Det[d]

4

RowReduce[r]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Kern, Im

NullSpace[r]

{}

RowReduce[r]

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

RowReduce[r].{x1,x2,x3}

{x1, x2, x3}

Inverse Matrix

rI=Inverse[r]

$\left\{\left\{\frac{11}{14}, -\frac{3}{7}, \frac{3}{14}\right\}, \left\{-\frac{1}{7}, \frac{5}{7}, -\frac{3}{28}\right\}, \left\{0, 0, \frac{1}{2}\right\}\right\}$

rI//MatrixForm

$$\begin{pmatrix} \frac{11}{14} & -\frac{3}{7} & \frac{3}{14} \\ -\frac{1}{7} & \frac{5}{7} & -\frac{3}{28} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

rI=mT.d.m1

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, \frac{3}{14}\right\}, \{0, 0, 2\}\right\}$$

m1.Inverse[d].mT

$$\left\{\left\{\frac{11}{14}, \frac{3}{7}, 0\right\}, \left\{\frac{1}{7}, \frac{5}{7}, 0\right\}, \left\{0, 0, \frac{1}{2}\right\}\right\}$$

m1.Inverse[d].mT//MatrixForm

$$\begin{pmatrix} \frac{11}{14} & \frac{3}{7} & 0 \\ \frac{1}{7} & \frac{5}{7} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

EW, EV

eW=Eigenvalues[r]

$$\{2, 2, 1\}$$

eV=Eigenvectors[r]

$$\left\{\left\{-\frac{3}{4}, 0, 1\right\}, \left\{\frac{3}{2}, 1, 0\right\}, \{-2, 1, 0\}\right\}$$

Eigenvectors[r]//MatrixForm

$$\begin{pmatrix} -\frac{3}{4} & 0 & 1 \\ \frac{3}{2} & 1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

Eigensystem[r]

$$\left\{\{2, 2, 1\}, \left\{\left\{-\frac{3}{4}, 0, 1\right\}, \left\{\frac{3}{2}, 1, 0\right\}, \{-2, 1, 0\}\right\}\right\}$$

Abbildung:

r.{x1,x2,x3}//MatrixForm

$$\begin{pmatrix} \frac{10\,x1}{7} + \frac{6\,x2}{7} - \frac{3\,x3}{7} \\ \frac{2\,x1}{7} + \frac{11\,x2}{7} + \frac{3\,x3}{14} \\ 2\,x3 \end{pmatrix}$$

{a x1,b x2, c x3}//MatrixForm

$$\begin{pmatrix} 2\,x1 & -x1 & 0 \\ 3\,x2 & 2\,x2 & 0 \\ 0 & 2\,x3 & 4\,x3 \end{pmatrix}$$

r.{1,0,0}

$$\left\{\frac{10}{7}, \frac{2}{7}, 0\right\}$$

r.{0,1,0}

$$\left\{\frac{6}{7}, \frac{11}{7}, 0\right\}$$

r.{0,0,1}

$$\left\{-\frac{3}{7}, \frac{3}{14}, 2\right\}$$

rI.{1,0,0}

$$\left\{\frac{10}{7}, \frac{2}{7}, 0\right\}$$

rI.{0,1,0}

$$\left\{\frac{6}{7}, \frac{11}{7}, 0\right\}$$

rI.{0,0,1}

$$\left\{-\frac{3}{7}, \frac{3}{14}, 2\right\}$$

r.eV[[1]]

$$\left\{-\frac{3}{2}, 0, 2\right\}$$

r.eV[[2]]

{3, 2, 0}

r.eV[[3]]

{-2, 1, 0}

rI.eV[[1]]

$$\left\{-\frac{3}{2}, 0, 2\right\}$$

rI.eV[[2]]

{3, 2, 0}

rI.eV[[3]]

{-2, 1, 0}

r.Transpose[{k1 eV[[1]],k2 eV[[2]],k3 eV[[3]]}]

$$\left\{\left\{-\frac{3\,k1}{2}, 3\,k2, -2\,k3\right\}, \{0, 2\,k2, k3\}, \{2\,k1, 0, 0\}\right\}$$

```
rI.Transpose[{k1 eV[[1]],k2 eV[[2]],k3 eV[[3]]}]
```

$$\left\{\left\{-\frac{3\,k1}{2},\ 3\,k2,\ -2\,k3\right\},\ \{0,\ 2\,k2,\ k3\},\ \{2\,k1,\ 0,\ 0\}\right\}$$

Uebung 3

Konstruktion der Matrix

```
eW1=1;eW2=2; eW3=0;
d={{eW1,0,0},{0,eW2,0},{0,0,eW3}};
a={2,-1,0}; b={3,2,0}; c={0,2,4};
m={a,b,c};
mT=Transpose[m];
m1=Inverse[mT];
r=mT.d.m1
```

$$\left\{\left\{\frac{10}{7},\ \frac{6}{7},\ -\frac{3}{7}\right\},\ \left\{\frac{2}{7},\ \frac{11}{7},\ -\frac{11}{14}\right\},\ \{0,\ 0,\ 0\}\right\}$$

```
r//MatrixForm
```

$$\begin{pmatrix} \frac{10}{7} & \frac{6}{7} & -\frac{3}{7} \\ \frac{2}{7} & \frac{11}{7} & -\frac{11}{14} \\ 0 & 0 & 0 \end{pmatrix}$$

```
mT//MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 0 \\ -1 & 2 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

```
MatrixPower[r,1]
```

$$\left\{\left\{\frac{10}{7},\ \frac{6}{7},\ -\frac{3}{7}\right\},\ \left\{\frac{2}{7},\ \frac{11}{7},\ -\frac{11}{14}\right\},\ \{0,\ 0,\ 0\}\right\}$$

```
MatrixPower[r,2]
```

$$\left\{\left\{\frac{16}{7},\ \frac{18}{7},\ -\frac{9}{7}\right\},\ \left\{\frac{6}{7},\ \frac{19}{7},\ -\frac{19}{14}\right\},\ \{0,\ 0,\ 0\}\right\}$$

```
mT.d.d.m1
```

$$\left\{\left\{\frac{16}{7},\ \frac{18}{7},\ -\frac{9}{7}\right\},\ \left\{\frac{6}{7},\ \frac{19}{7},\ -\frac{19}{14}\right\},\ \{0,\ 0,\ 0\}\right\}$$

Daten

```
MatrixRank[r]
```

2

NullSpace[r]

$$\left\{\left\{0, \frac{1}{2}, 1\right\}\right\}$$

CharacteristicPolynomial[r,x]

$$-2\,x + 3\,x^2 - x^3$$

CharacteristicPolynomial[d,x]

$$-2\,x + 3\,x^2 - x^3$$

Tr[r]

3

Tr[d]

3

Det[r]

0

Det[d]

0

RowReduce[r]

$$\left\{\{1, 0, 0\}, \left\{0, 1, -\frac{1}{2}\right\}, \{0, 0, 0\}\right\}$$

Kern, Im

NullSpace[r]

$$\left\{\left\{0, \frac{1}{2}, 1\right\}\right\}$$

RowReduce[r]

$$\left\{\{1, 0, 0\}, \left\{0, 1, -\frac{1}{2}\right\}, \{0, 0, 0\}\right\}$$

RowReduce[r].{x1,x2,x3}

$$\left\{x1, x2 - \frac{x3}{2}, 0\right\}$$

Inverse Matrix

rI=Inverse[r]

Inverse::sing : Matrix $\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, -\frac{11}{14}\right\}, \{0, 0, 0\}\right\}$ is singular. Mehr…

$$\text{Inverse}\left[\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, -\frac{11}{14}\right\}, \{0, 0, 0\}\right\}\right]$$

rI//MatrixForm

$$\text{Inverse}\left[\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, -\frac{11}{14}\right\}, \{0, 0, 0\}\right\}\right]$$

rI=mT.d.m1

$$\left\{\left\{\frac{10}{7}, \frac{6}{7}, -\frac{3}{7}\right\}, \left\{\frac{2}{7}, \frac{11}{7}, -\frac{11}{14}\right\}, \{0, 0, 0\}\right\}$$

m1.Inverse[d].mT

```
Inverse::sing : Matrix {{1, 0, 0}, {0, 2, 0}, {0, 0, 0}} is singular. Mehr…
```

$$\left\{\left\{\frac{2}{7}, -\frac{3}{7}, \frac{3}{14}\right\}, \left\{\frac{1}{7}, \frac{2}{7}, -\frac{1}{7}\right\}, \left\{0, 0, \frac{1}{4}\right\}\right\}.$$
$$\text{Inverse}[\{\{1, 0, 0\}, \{0, 2, 0\}, \{0, 0, 0\}\}].\{\{2, 3, 0\}, \{-1, 2, 2\}, \{0, 0, 4\}\}$$

m1.Inverse[d].mT//MatrixForm

```
Inverse::sing : Matrix {{1, 0, 0}, {0, 2, 0}, {0, 0, 0}} is singular. Mehr…
```

$$\left\{\left\{\frac{2}{7}, -\frac{3}{7}, \frac{3}{14}\right\}, \left\{\frac{1}{7}, \frac{2}{7}, -\frac{1}{7}\right\}, \left\{0, 0, \frac{1}{4}\right\}\right\}.$$
$$\text{Inverse}[\{\{1, 0, 0\}, \{0, 2, 0\}, \{0, 0, 0\}\}].\{\{2, 3, 0\}, \{-1, 2, 2\}, \{0, 0, 4\}\}$$

EW, EV

eW=Eigenvalues[r]

$$\{2, 1, 0\}$$

eV=Eigenvectors[r]

$$\left\{\left\{\frac{3}{2}, 1, 0\right\}, \{-2, 1, 0\}, \left\{0, \frac{1}{2}, 1\right\}\right\}$$

Eigenvectors[r]//MatrixForm

$$\begin{pmatrix} \frac{3}{2} & 1 & 0 \\ -2 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{pmatrix}$$

Eigensystem[r]

$$\left\{\{2, 1, 0\}, \left\{\left\{\frac{3}{2}, 1, 0\right\}, \{-2, 1, 0\}, \left\{0, \frac{1}{2}, 1\right\}\right\}\right\}$$

Abbildung:

r.{x1,x2,x3}//MatrixForm

$$\begin{pmatrix} \frac{10\,x1}{7} + \frac{6\,x2}{7} - \frac{3\,x3}{7} \\ \frac{2\,x1}{7} + \frac{11\,x2}{7} - \frac{11\,x3}{14} \\ 0 \end{pmatrix}$$

{a x1,b x2, c x3}//MatrixForm

$$\begin{pmatrix} 2\,x1 & -x1 & 0 \\ 3\,x2 & 2\,x2 & 0 \\ 0 & 2\,x3 & 4\,x3 \end{pmatrix}$$

r.{1,0,0}

$$\left\{\frac{10}{7}, \frac{2}{7}, 0\right\}$$

r.{0,1,0}

$$\left\{\frac{6}{7}, \frac{11}{7}, 0\right\}$$

r.{0,0,1}

$$\left\{-\frac{3}{7}, -\frac{11}{14}, 0\right\}$$

rI.{1,0,0}

$$\left\{\frac{10}{7}, \frac{2}{7}, 0\right\}$$

rI.{0,1,0}

$$\left\{\frac{6}{7}, \frac{11}{7}, 0\right\}$$

rI.{0,0,1}

$$\left\{-\frac{3}{7}, -\frac{11}{14}, 0\right\}$$

r.eV[[1]]

{3, 2, 0}

r.eV[[2]]

{−2, 1, 0}

r.eV[[3]]

{0, 0, 0}

rI.eV[[1]]

{3, 2, 0}

rI.eV[[2]]

{−2, 1, 0}

rI.eV[[3]]

{0, 0, 0}

r.Transpose[{k1 eV[[1]],k2 eV[[2]],k3 eV[[3]]}]

{{3 k1, −2 k2, 0}, {2 k1, k2, 0}, {0, 0, 0}}

```
rI.Transpose[{k1 eV[[1]],k2 eV[[2]],k3 eV[[3]]}]
```

```
{{3 k1, -2 k2, 0}, {2 k1, k2, 0}, {0, 0, 0}}
```