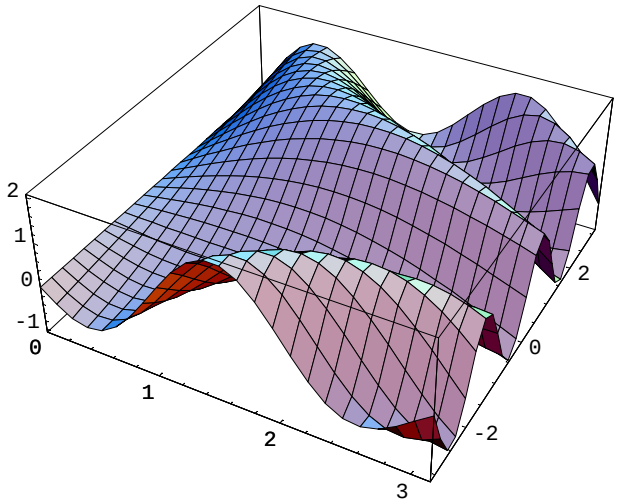
Lösungen

1

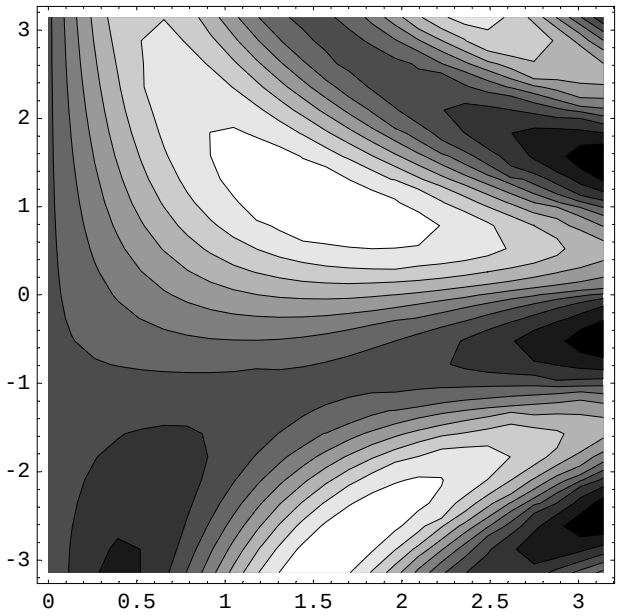
```
f[x_,y_]:=Sin[x y]+Sin[x];
f[{x_,y_}]:=f[x,y];
```

a

```
Plot3D[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```




```
ContourPlot[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```



b

grad[f_,x_,y_]:={D[f,x],D[f,y]};
grad[f[x,y],x,y]

$$\{\text{Cos}[x] + y\,\text{Cos}[x\,y],\ x\,\text{Cos}[x\,y]\}$$

Solve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is $-\text{ArcCos}[\text{Cos}[x]] == 0$. Mehr…

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is $\text{ArcCos}[\text{Cos}[x\,y]] == 0$. Mehr…

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…

$$\left\{\{y \to -1,\ x \to 0\},\ \left\{y \to 1,\ x \to -\frac{\pi}{2}\right\},\ \left\{y \to 1,\ x \to \frac{\pi}{2}\right\}\right\}$$

NSolve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is $-\text{ArcCos}[\text{Cos}[x]] == 0$. Mehr…

Solve::incnst : Inconsistent or redundant transcendental equation. After reduction, the bad equation is $\text{ArcCos}[\text{Cos}[x\,y]] == 0$. Mehr…

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…

$$\{\{y \to 1.,\ x \to -1.5708\},\ \{y \to 1.,\ x \to 1.5708\}\}$$

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0}]

$$\{x \to 0.,\ y \to -1.\}$$

f[0,-1]

0

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0.5}]

$$\{x \to 1.5708,\ y \to 1.\}$$

f[Pi/2,1]

2

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,3}]

$$\{x \to 1.5708,\ y \to 3.\}$$

f[Pi/2,3]

0

Rand

Evaluate[(D[f[x,y],y]==0)/.x->0]

```
True
```

Alles Lösungen

Evaluate[(D[f[x,y],y]==0)/.x->Pi]

$$\pi \text{Cos}[\pi y] == 0$$

Solve[Evaluate[(D[f[x,y],y]==0)/.x->Pi],{y}]

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

$$\left\{\left\{y \to -\frac{1}{2}\right\}, \left\{y \to \frac{1}{2}\right\}\right\}$$

Evaluate[(D[f[x,y],x]==0)/.y->-Pi]

$$\text{Cos}[x] - \pi \text{Cos}[\pi x] == 0$$

Solve[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x}]

```
Solve::tdep : The equations appear to involve the
   variables to be solved for in an essentially non-algebraic way. Mehr…
```

$$\text{Solve}[\text{Cos}[x] - \pi \text{Cos}[\pi x] == 0, \{x\}]$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,0}]

```
FindRoot::jsing :
 Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr…
```

$$\{x \to 0.\}$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,0.1}]

$$\{x \to 0.405515\}$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,1}]

$$\{x \to 5.57793\}$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,1.1}]

$$\{x \to 1.50651\}$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,2}]

$$\{x \to -1.50651\}$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,2.1}]

$$\{x \to 3.40044\}$$

Evaluate[(D[f[x,y],x]==0)/.y->Pi]

$$\text{Cos}[x] + \pi \text{Cos}[\pi x] == 0$$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0}]

```
FindRoot::jsing :
 Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr…
```

$\{x \to 0.\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.05}]

$\{x \to 2.42298\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.1}]

$\{x \to 1.49203\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,1}]

$\{x \to -2.42298\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.5}]

$\{x \to 0.58547\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2}]

$\{x \to 3.59249\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2.1}]

$\{x \to 2.42298\}$

c

grad[f[x,y],x,y].{1,2} 1/Norm[{1,2}]/.{x->1,y->1}

$$\frac{4\,\mathrm{Cos}[1]}{\sqrt{5}}$$

N[%]

0.966522

d

ArcTan[%]

0.768376

%/Degree

44.0247

e

g[x_,y_]:=y^2-x

Solve[Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y],g[x,y]==0}],{x,y,λ}]

```
Solve::tdep : The equations appear to involve the
   variables to be solved for in an essentially non-algebraic way. Mehr…
```

```
Solve[{{Cos[x] + y Cos[x y], x Cos[x y]} == {-λ, 2 y λ}, -x + y^2 == 0}, {x, y, λ}]
```

Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y]}]/.x->y^2

```
{{Cos[y^2] + y Cos[y^3], y^2 Cos[y^3]} == {-λ, 2 y λ}}
```

Evaluate[grad[f[x,y],x,y][[2]]== λ grad[g[x,y],x,y][[2]]/.
λ->-grad[f[x,y],x,y][[1]]] /.x->y^2

```
y^2 Cos[y^3] == 2 y (-Cos[y^2] - y Cos[y^3])
```

InputForm[%]

```
y^2*Cos[y^3] == 2*y*(-Cos[y^2] - y*Cos[y^3])
```

```
tabl1=Table[FindRoot[y^2*Cos[y^3] == 2*y*(-Cos[y^2] - y*Cos[y^3]),{y,k
Pi}],{k,-1,1,0.0125}]//Flatten;tabl1=Union[tabl1//Chop];
tabl1//MatrixForm
```

$$\begin{pmatrix} y \to -3.93852 \\ y \to -3.09442 \\ y \to -2.9965 \\ y \to -2.86363 \\ y \to -2.72898 \\ y \to -2.59678 \\ y \to -2.40392 \\ y \to -2.40392 \\ y \to -2.40392 \\ y \to -2.22876 \\ y \to -2.22876 \\ y \to -2.00558 \\ y \to -1.63135 \\ y \to -1.63135 \\ y \to -1.11101 \\ y \to -1.11101 \\ y \to -1.11101 \\ y \to -0.634007 \\ y \to -0.634007 \\ y \to -0.634007 \\ y \to 0 \\ y \to 1.18504 \\ y \to 1.18504 \\ y \to 1.18504 \\ y \to 1.18504 \\ y \to 1.72169 \\ y \to 1.96582 \\ y \to 1.96582 \\ y \to 2.21938 \\ y \to 2.21938 \\ y \to 2.43266 \\ y \to 2.43266 \\ y \to 2.57288 \\ y \to 2.57288 \\ y \to 2.73715 \\ y \to 2.87038 \\ y \to 2.87038 \\ y \to 2.98174 \\ y \to 2.98174 \\ y \to 2.98174 \\ y \to 3.10912 \end{pmatrix}$$

```
tabl2=Union[Round[100000
Table[{u=y/.tabl1[[k]],u^2},{k,1,Length[tabl1]}]]/100000]//N;
tabl3=Map[Reverse,tabl2]  ;
tabl4 = Select[tabl3,#[[1]]<=Pi&]; tabl4//MatrixForm
```

$$\begin{pmatrix} 2.6613 & -1.63135 \\ 1.23435 & -1.11101 \\ 0.40196 & -0.63401 \\ 0. & 0. \\ 1.40431 & 1.18504 \\ 2.9642 & 1.72169 \end{pmatrix}$$

Extrema

```
{{2.6613   -1.63135},
 {1.23435  -1.11101},
 {0.40196  -0.63401},
 {0.       0.      },
 {1.40431  1.18504 },
 {2.9642   1.72169 }}
```

Funktionswerte in den Extrema

```
Map[f,tabl4]//MatrixForm
```

```
{{1.39405   },
 {-0.0362476},
 {0.139126  },
 {0.        },
 {1.98182   },
 {-0.748048 }}
```

2

```
Remove["Global`*"]

k=1;
f[g_,h_,exp_]:=1/(1/g+1/h+k 10^(exp)/(g+h)); f[g,h,exp]
```

$$\frac{1}{\frac{1}{g}+\frac{1}{h}+\frac{10^{exp}}{g+h}}$$

```
fehler[g_,h_,exp_,Δg_,Δh_]:= Abs[D[f[g,h,exp],g]] Δg + Abs[D[f[g,h,exp],h]] Δh;
fehler[g,h,exp,Δg,Δh]
```

$$\Delta g\ \text{Abs}\left[\frac{-\frac{1}{g^2}-\frac{10^{exp}}{(g+h)^2}}{\left(\frac{1}{g}+\frac{1}{h}+\frac{10^{exp}}{g+h}\right)^2}\right]+\Delta h\ \text{Abs}\left[\frac{-\frac{1}{h^2}-\frac{10^{exp}}{(g+h)^2}}{\left(\frac{1}{g}+\frac{1}{h}+\frac{10^{exp}}{g+h}\right)^2}\right]$$

```
f[g,h,exp]/. {g->14.28, h->25.62,exp->4}
```

```
0.00398826
```

```
fehler[g,h,exp,Δg,Δh]/. {g->14.28, h->25.62,exp->4,Δg->0.1,Δh->0.25}
```

```
0.0000349834
```

```
f[g,h,exp]/. {g->14.28, h->25.62,exp->-4}
```

```
9.16905
```

```
fehler[g,h,exp,Δg,Δh]/. {g->14.28, h->25.62,exp->-4,Δg->0.1,Δh->0.25}
```

```
0.0732506
```

```
k=0;
f[g_,h_,exp_]:=1/(1/g+1/h+k 10^(exp)/(g+h));

f[g,h,exp]/. {g->14.28, h->25.62,exp->-4}
```

```
9.16926
```

```
fehler[g,h,exp,Δg,Δh]/. {g->14.28, h->25.62,exp->-4,Δg->0.1,Δh->0.25}
```

```
0.0732521
```

Anders

```
h[x_,y_]:=1/(1/x+1/y+10^(-4)/(x+y))
```

```
Fehler= Abs[D[h[x,y],x]] 0.1+Abs[D[h[x,y],y]] 0.25
```

```
General::spell1 :
 Possible spelling error: new symbol name "Fehler" is similar to existing symbol "fehler". Mehr…
```

$$0.1\,\text{Abs}\left[\frac{-\frac{1}{x^2}-\frac{1}{10000\,(x+y)^2}}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{10000\,(x+y)}\right)^2}\right]+0.25\,\text{Abs}\left[\frac{-\frac{1}{y^2}-\frac{1}{10000\,(x+y)^2}}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{10000\,(x+y)}\right)^2}\right]$$

```
Fehler= Abs[D[h[x,y],x]] 0.1+Abs[D[h[x,y],y]] 0.25 /.{x->14.28,y->25.62}
```

```
0.0732506
```

```
h[x,y]/.{x->14.28,y->25.62}
```

```
9.16905
```

```
h[x_,y_]:=1/(1/x+1/y+10^(4)/(x+y))
```

```
Fehler= Abs[D[h[x,y],x]] 0.1+Abs[D[h[x,y],y]] 0.25
```

$$0.1\,\text{Abs}\left[\frac{-\frac{1}{x^2}-\frac{10000}{(x+y)^2}}{\left(\frac{1}{x}+\frac{1}{y}+\frac{10000}{x+y}\right)^2}\right]+0.25\,\text{Abs}\left[\frac{-\frac{1}{y^2}-\frac{10000}{(x+y)^2}}{\left(\frac{1}{x}+\frac{1}{y}+\frac{10000}{x+y}\right)^2}\right]$$

```
Fehler= Abs[D[h[x,y],x]] 0.1+Abs[D[h[x,y],y]] 0.25 /.{x->14.28,y->25.62}
```

```
0.0000349834
```

```
h[x,y]/.{x->14.28,y->25.62}
```

```
0.00398826
```

3

```
Remove["Global`*"]
```

a

```
f[x_,a_,b_,c_,f_]:=(a x^2+b x+c)/(x^2+f)-2;
Solve[{f[x,a,b,c,f]==f[-x,a,b,c,f]},{b}]
```

```
{{b → 0}}
```

Solve[{f[x,a,0,c,f]==f[-x,a,0,c,f],f[0,a,0,c,f]==1,f[1,a,0,c,f]==3/2},{a,c,f}]

```
Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…
```

$$\left\{\left\{a \to \frac{7}{2} + \frac{f}{2},\ c \to 3\,f\right\}\right\}$$

f[x,7/2+f/2,0,3f,f]//Simplify

$$\frac{3\,x^2 + f\,(2 + x^2)}{2\,(f + x^2)}$$

Limit[f[x,7/2+f/2,0,3f,f],x->Infinity]==2

$$\frac{3 + f}{2} == 2$$

f[x,7/2+1/2,0,3 1,1]

$$-2 + \frac{3 + 4\,x^2}{1 + x^2}$$

D[f[x,7/2+1/2,0,3 1,1],x]

$$\frac{8\,x}{1 + x^2} - \frac{2\,x\,(3 + 4\,x^2)}{(1 + x^2)^2}$$

(D[f[x,7/2+1/2,0,3 1,1],x]/.x->1)==1/2

```
True
```

b

Integrate[f[x,7/2+1/2,0,3 1,1],{x,-2,2}]

$$8 - 2\,\text{ArcTan}[2]$$

N[%]

```
5.7857
```

c

Integrate[Pi (f[2,7/2+1/2,0,3 1,1]^2-f[x,7/2+1/2,0,3 1,1]^2),{x,-2,2}]

$$\pi\left(-\frac{86}{25} + 7\,\text{ArcTan}[2]\right)$$

N[%]

```
13.5404
```