Lösungen

1

```
Remove["Global`*"]
```

a

```
f[x_]:=Sin[x];
k[x_]:=Evaluate[(f''[x])/(1+(f'[x])^2)^(3/2)]; k[x]
```

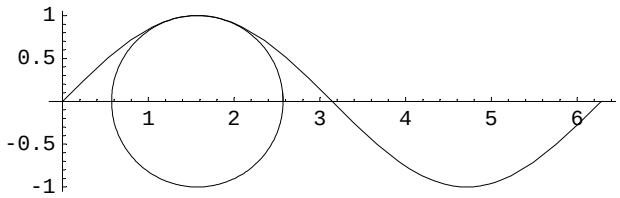
$$-\frac{\text{Sin}[x]}{(1+\text{Cos}[x]^2)^{3/2}}$$

```
r[x_]:=Abs[1/k[x]]; r[x]/.x->Pi/2
```

```
1
```

b

```
Plot[f[x],{x,0,2Pi},AspectRatio->Automatic, Epilog->{Circle[{Pi/2,0},1]}];
```



c

Auf der x-Achse bei Pi/2

2

```
Remove["Global`*"]
```

a

```
m[x_]:= -q (zL-x)^2 / 2; m[x]
```

$$-\frac{1}{2}\,q\,(-x+zL)^2$$

```
s[x_] := DSolve[y''[x] == -m[x] / zE / zI, y[x], x] // Flatten; s[x]
```

$$\left\{y[x] \to \frac{q\,(x - zL)^4}{24\,zE\,zI} + C[1] + x\,C[2]\right\}$$

```
s[z]
```

$$\left\{y[z] \to \frac{q\,(z - zL)^4}{24\,zE\,zI} + C[1] + z\,C[2]\right\}$$

```
s[z][[1]][[2]] //FullForm
```

```
Plus[Times[Rational[1, 24], q, Power[zE, -1],
  Power[zI, -1], Power[Plus[z, Times[-1, zL]], 4]], C[1], Times[z, C[2]]]
```

```
s[z][[1]][[2]] /. {C[1]->C1, C[2]->C2}
```

$$C1 + C2\,z + \frac{q\,(z - zL)^4}{24\,zE\,zI}$$

```
h[z_]:=s[z][[1]][[2]] /. {C[1]->C1, C[2]->C2} ; h[z]
```

$$C1 + C2\,z + \frac{q\,(z - zL)^4}{24\,zE\,zI}$$

```
h[3]
```

$$\frac{q\,(-3 + zL)^2}{2\,zE\,zI}$$

```
h[z]/.z->3
```

$$C1 + 3\,C2 + \frac{q\,(3 - zL)^4}{24\,zE\,zI}$$

```
h1[u_]:=h[z]/.z->u;  h1[3]
```

$$C1 + 3\,C2 + \frac{q\,(3 - zL)^4}{24\,zE\,zI}$$

```
solv1=Solve[{h1[0]==0,Evaluate[h1'[u]==0]/.u->0},{C1,C2}]//Simplify//Flatten
```

$$\left\{C1 \to -\frac{q\,zL^4}{24\,zE\,zI},\ C2 \to \frac{q\,zL^3}{6\,zE\,zI}\right\}$$

```
h2[u_]:=(h1[u]/.solv1)//Simplify;
h3[t_]=InputForm[h2[t]]; {h2[u],h3[z]}
```

$$\left\{\frac{q\,u^2\,(u^2 - 4\,u\,zL + 6\,zL^2)}{24\,zE\,zI},\ (q*z\text{^}2*(z\text{^}2 - 4*z*zL + 6*zL\text{^}2))/(24*zE*zI)\right\}$$

```
FullForm[h3[t]]
```

```
InputForm[Times[Rational[1, 24], q, Power[t, 2], Power[zE, -1],
  Power[zI, -1], Plus[Power[t, 2], Times[-4, t, zL], Times[6, Power[zL, 2]]]]]
```

b

Masse in Meter und kg

hoehe= 0.05 Meter; A=hoehe^2; zI = A hoehe^2 /12

$5.20833 \times 10^{-7}\ \text{Meter}^4$

h3[u]

```
(79999.99999999996*q*u^2*(u^2 - 4*u*zL + 6*zL^2))/(Meter^4*zE)
```

A

$0.0025\ \text{Meter}^2$

zE=210000 (10^3)^2 kg Meter/Sek^2 /Meter^2

$$\frac{210000000000\ \text{kg}}{\text{Meter}\ \text{Sek}^2}$$

zL = 2 Meter

$2\ \text{Meter}$

q = 700 9.81 /zL kg Meter/Sek^2

$$\frac{3433.5\ \text{kg}}{\text{Sek}^2}$$

h3[t]

```
(0.0013079999999999993*t^2*(24*Meter^2 - 8*Meter*t + t^2))/Meter^3
```

h3[2 Meter]

```
0.06278399999999996*Meter
```

c

q zL^4/zE/zI (* Einheitenvergleich *)

$0.502272\ \text{Meter}$

3

Remove["Global`*"]

a

v1={1,0,1}; w1={2,0,0}; v2={-1,1,0}; w2={0,1,2}; v3={0,-1,2}; w3={-1,1,3};
A= Transpose[{w1,w2,w3}].Inverse[Transpose[{v1,v2,v3}]]; A//MatrixForm

$$\begin{pmatrix} \frac{5}{3} & \frac{5}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{5}{3} & \frac{1}{3} & \frac{5}{3} \end{pmatrix}$$

b

```
A.Transpose[{{1,1,1}}]// MatrixForm
```

$$\begin{pmatrix} \frac{11}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

c

```
Det[A]
```

$$\frac{2}{3}$$

4

```
Remove["Global`*"]
```

a

```
a={1,1,2}; b={-1,1,1}; u=Cross[a,b]
```

```
{-1, -3, 2}
```

```
M= Transpose[{{0,0,0},a,b}].Inverse[Transpose[{u,a,b}]]; M//MatrixForm
```

$$\begin{pmatrix} \frac{13}{14} & -\frac{3}{14} & \frac{1}{7} \\ -\frac{3}{14} & \frac{5}{14} & \frac{3}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{5}{7} \end{pmatrix}$$

```
dM= {{Cos[Pi/6],-Sin[Pi/6],0},{Sin[Pi/6],Cos[Pi/6],0},{0,0,1}}; dM//MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Abb=dM.M; Abb//MatrixForm
```

$$\begin{pmatrix} \frac{3}{28}+\frac{13\sqrt{3}}{28} & -\frac{5}{28}-\frac{3\sqrt{3}}{28} & -\frac{3}{14}+\frac{\sqrt{3}}{14} \\ \frac{13}{28}-\frac{3\sqrt{3}}{28} & -\frac{3}{28}+\frac{5\sqrt{3}}{28} & \frac{1}{14}+\frac{3\sqrt{3}}{14} \\ \frac{1}{7} & \frac{3}{7} & \frac{5}{7} \end{pmatrix}$$

```
dM.M//N//MatrixForm
```

$$\begin{pmatrix} 0.911309 & -0.364148 & -0.0905678 \\ 0.278709 & 0.202152 & 0.442582 \\ 0.142857 & 0.428571 & 0.714286 \end{pmatrix}$$

b

```
dM.M.Transpose[{{1,-1,-1}}]//MatrixForm
```

$$\begin{pmatrix} \frac{1}{2}+\frac{\sqrt{3}}{2} \\ \frac{1}{2}-\frac{\sqrt{3}}{2} \\ -1 \end{pmatrix}$$

```
dM.M.Transpose[{{1,-1,-1}}]//N//MatrixForm
```

$$\begin{pmatrix} 1.36603 \\ -0.366025 \\ -1. \end{pmatrix}$$

c

```
Eigenvalues[Abb]
```

$$\left\{\frac{1}{28}\left(10+9\sqrt{3}+i\sqrt{-\left(-10-9\sqrt{3}\right)^2+56\left(4+5\sqrt{3}\right)}\right),\right.$$
$$\left.\frac{1}{28}\left(10+9\sqrt{3}-i\sqrt{-\left(-10-9\sqrt{3}\right)^2+56\left(4+5\sqrt{3}\right)}\right),0\right\}$$

```
Eigenvalues[Abb]//N
```

$$\{0.913873+0.262943\,i,\ 0.913873-0.262943\,i,\ 0.\}$$

---

5 Nicht erfüllbare Erwartungen

---

5 (a) Fall der Hauptspannungsrichtungen "{-3,10,5}, {0,3,-6}"

```
Remove["Global`*"]
```

a

```
a={-3,10,5}; b={0,3,-6}; a.b
```

```
0
```

```
0
```

```
a1=(A.Transpose[{1/5 {0,3,4}}]==Transpose[{{4000,900,-1200}}])
```

$$A.\left\{\{0\},\left\{\frac{3}{5}\right\},\left\{\frac{4}{5}\right\}\right\} == \{\{4000\},\{900\},\{-1200\}\}$$

$$A.\left\{\{0\},\left\{\frac{3}{5}\right\},\left\{\frac{4}{5}\right\}\right\} == \{\{4000\},\{900\},\{-1200\}\}$$

a2=(A.Transpose[{a}]==3000 Transpose[{a}])

$$A.\{\{-3\},\{10\},\{5\}\} == \{\{-9000\},\{30000\},\{15000\}\}$$

$$A.\{\{-3\},\{10\},\{5\}\} == \{\{-9000\},\{30000\},\{15000\}\}$$

a3=(A.Transpose[{b}]==0 Transpose[{b}])

$$A.\{\{0\},\{3\},\{-6\}\} == \{\{0\},\{0\},\{0\}\}$$

$$A.\{\{0\},\{3\},\{-6\}\} == \{\{0\},\{0\},\{0\}\}$$

A.Transpose[{1/5 {0,3,4},a,b}]==Transpose[{{4000,900,-1200},3000 a,0 b}]

$$A.\left\{\{0,-3,0\},\left\{\frac{3}{5},10,3\right\},\left\{\frac{4}{5},5,-6\right\}\right\} == \{\{4000,-9000,0\},\{900,30000,0\},\{-1200,15000,0\}\}$$

$$A.\left\{\{0,-3,0\},\left\{\frac{3}{5},10,3\right\},\left\{\frac{4}{5},5,-6\right\}\right\} == \{\{4000,-9000,0\},\{900,30000,0\},\{-1200,15000,0\}\}$$

A=Transpose[{{4000,900,-1200},3000 a,0 b}].Inverse[Transpose[{1/5 {0,3,4}, a, b}]]; A//MatrixForm

$$\begin{pmatrix} \frac{59000}{3} & 4000 & 2000 \\ -6250 & 900 & 450 \\ -10000 & -1200 & -600 \end{pmatrix}$$

$$\begin{pmatrix} \frac{59000}{3} & 4000 & 2000 \\ -6250 & 900 & 450 \\ -10000 & -1200 & -600 \end{pmatrix}$$

A//N//MatrixForm

$$\begin{pmatrix} 19666.7 & 4000. & 2000. \\ -6250. & 900. & 450. \\ -10000. & -1200. & -600. \end{pmatrix}$$

$$\begin{pmatrix} 19666.7 & 4000. & 2000. \\ -6250. & 900. & 450. \\ -10000. & -1200. & -600. \end{pmatrix}$$

Mit obigen Angaben wird die Transformationsmatrix nicht-symmetrisch. Der in der Aufgabe gegebene Spannungsvektor kann so nicht gegeben werden.

b

n3=Cross[a,b]

$$\{-75,-18,-9\}$$

$$\{-75,-18,-9\}$$

```
n2Norm=1/Norm[n3] n3
```

$$\left\{-5\sqrt{\frac{5}{134}}, -3\sqrt{\frac{2}{335}}, -\frac{3}{\sqrt{670}}\right\}$$

$$\left\{-5\sqrt{\frac{5}{134}}, -3\sqrt{\frac{2}{335}}, -\frac{3}{\sqrt{670}}\right\}$$

```
%//N
```

$$\{-0.965834, -0.2318, -0.1159\}$$

$$\{-0.965834, -0.2318, -0.1159\}$$

c 3. Hauptspannungsrichtungsvektor nicht Eigenvektor ==> Hauptspannung als Eigenwert nicht berechenbar

```
{n3/(-6),a/4,b/6}
```

$$\left\{\left\{\frac{25}{2}, 3, \frac{3}{2}\right\}, \left\{-\frac{3}{4}, \frac{5}{2}, \frac{5}{4}\right\}, \left\{0, \frac{1}{2}, -1\right\}\right\}$$

$$\left\{\left\{\frac{25}{2}, 3, \frac{3}{2}\right\}, \left\{-\frac{3}{4}, \frac{5}{2}, \frac{5}{4}\right\}, \left\{0, \frac{1}{2}, -1\right\}\right\}$$

```
Eigensystem[A]
```

$$\left\{\left\{\frac{50900}{3}, 3000, 0\right\}, \left\{\left\{-\frac{5090}{2757}, \frac{2743}{3676}, 1\right\}, \left\{-\frac{3}{5}, 2, 1\right\}, \left\{0, -\frac{1}{2}, 1\right\}\right\}\right\}$$

$$\left\{\left\{\frac{50900}{3}, 3000, 0\right\}, \left\{\left\{-\frac{5090}{2757}, \frac{2743}{3676}, 1\right\}, \left\{-\frac{3}{5}, 2, 1\right\}, \left\{0, -\frac{1}{2}, 1\right\}\right\}\right\}$$

```
%//N
```

$$\{\{16966.7, 3000., 0.\}, \{\{-1.84621, 0.746192, 1.\}, \{-0.6, 2., 1.\}, \{0., -0.5, 1.\}\}\}$$

$$\{\{16966.7, 3000., 0.\}, \{\{-1.84621, 0.746192, 1.\}, \{-0.6, 2., 1.\}, \{0., -0.5, 1.\}\}\}$$

---

5 (b) Fall der Hauptspannungsrichtungen "{-3,10,4}, {0,2,6}"

```
Remove["Global`*"]
```

a

```
a = {-3,10,4}; b = {0,2,6};
a.b
```

44

Problem: Hauptspannungsrichtungen nicht senkrecht zueinander

```
a1=(A.Transpose[{1/5 {0,3,4}}]==Transpose[{{4000,900,-1200}}])
```

$$A.\left\{\{0\}, \left\{\frac{3}{5}\right\}, \left\{\frac{4}{5}\right\}\right\} == \{\{4000\}, \{900\}, \{-1200\}\}$$

a2=(A.Transpose[{a}]==3000 Transpose[{a}])

$$A.\{\{-3\},\{10\},\{4\}\} == \{\{-9000\},\{30000\},\{12000\}\}$$

a3=(A.Transpose[{b}]==0 Transpose[{b}])

$$A.\{\{0\},\{2\},\{6\}\} == \{\{0\},\{0\},\{0\}\}$$

A.Transpose[{1/5 {0,3,4},a,b}]==Transpose[{{4000,900,-1200},3000 a,0 b}]

$$A.\left\{\{0,-3,0\},\left\{\frac{3}{5},10,2\right\},\left\{\frac{4}{5},4,6\right\}\right\} == \{\{4000,-9000,0\},\{900,30000,0\},\{-1200,12000,0\}\}$$

A=Transpose[{{4000,900,-1200},3000 a,0 b}].Inverse[Transpose[{1/5 {0,3,4}, a, b}]]; A//MatrixForm

$$\begin{pmatrix} \frac{113000}{3} & 12000 & -4000 \\ -2200 & 2700 & -900 \\ -14400 & -3600 & 1200 \end{pmatrix}$$

A//N//MatrixForm

$$\begin{pmatrix} 37666.7 & 12000. & -4000. \\ -2200. & 2700. & -900. \\ -14400. & -3600. & 1200. \end{pmatrix}$$

Mit obigen Angaben wird die Transformationsmatrix nicht-symmetrisch. Der in der Aufgabe gegebene Spannungsvektor kann so nicht gegeben werden. Zudem sind noch die gegebenen Hauptspannungsvektoren nicht orthogonal.

b

n3=Cross[a,b]

$$\{52, 18, -6\}$$

n2Norm=1/Norm[n3] n3

$$\left\{13\sqrt{\frac{2}{383}}, \frac{9}{\sqrt{766}}, -\frac{3}{\sqrt{766}}\right\}$$

%//N

$$\{0.939418, 0.325183, -0.108394\}$$

c 3. Hauptspannungsrichtungsvektor nicht Eigenvektor ==> Hauptspannung als Eigenwert nicht berechenbar

{n3/(-6),a/4,b/6}

$$\left\{\left\{-\frac{26}{3}, -3, 1\right\}, \left\{-\frac{3}{4}, \frac{5}{2}, 1\right\}, \left\{0, \frac{1}{3}, 1\right\}\right\}$$

Eigensystem[A]

$$\left\{\left\{\frac{115700}{3}, 3000, 0\right\}, \left\{-\frac{11570}{4401}, \frac{799}{5868}, 1\right\}, \left\{-\frac{3}{4}, \frac{5}{2}, 1\right\}, \left\{0, \frac{1}{3}, 1\right\}\right\}$$

%//N

```
{{38566.7, 3000., 0.},
 {{-2.62895, 0.136162, 1.}, {-0.75, 2.5, 1.}, {0., 0.333333, 1.}}}
```