Lösungen

1

```
Remove["Global`*"]
```

```
M.(Id - X).Inverse[M] + M - A.M == A.Transpose[M] - 3 M
```

```
M - A.M + M.(Id - X).Inverse[M] == -3 M + A.Transpose[M]
```

```
Id - M.X.Inverse[M] + M - A.M == A.Transpose[M] - 3 M
```

```
Id + M - A.M - M.X.Inverse[M] == -3 M + A.Transpose[M]
```

```
Id - A.Transpose[M] + 3 M - A.M + M == M.X.Inverse[M]
```

```
Id + 4 M - A.M - A.Transpose[M] == M.X.Inverse[M]
```

```
X == Inverse[M].(Id - A.Transpose[M] + 4 M - A.M).M
```

```
X == Inverse[M].(Id + 4 M - A.M - A.Transpose[M]).M
```

```
X == Id - Inverse[M].A.Transpose[M].M + 4 M - Inverse[M].A.M.M
```

```
X == Id + 4 M - Inverse[M].A.M.M - Inverse[M].A.Transpose[M].M
```

2

```
Remove["Global`*"]
```

```
r0 = {1, 2, -1};
a = {2, 1, 1};
q = {3, 10, 14};
rq = q - r0
```

```
{2, 8, 15}
```

```
gq = Norm[Cross[a, rq]] / Norm[a]
```

$$7\sqrt{\frac{7}{2}}$$

```
N[%]
```

```
13.0958
```

3

```
Remove["Global`*"]
```

```
r0 = {1, 2, -1};
a = {2, 1, 1};
b = {1, -1, -2};
q = {3, 10, 14};
rq = q - r0
```

```
{2, 8, 15}
```

a

```
gq = -Det[{a, b, rq}] / Norm[Cross[a, b]]
```

$$\sqrt{\frac{7}{5}}$$

```
N[%]
```

```
1.18322
```

b

```
v[λ_, μ_] := r0 + λ a + μ b; v[λ, μ]
```

$$\{1 + 2\,\lambda + \mu,\ 2 + \lambda - \mu,\ -1 + \lambda - 2\,\mu\}$$

```
w[t_] := q + t Cross[a, b]; w[t]
```

$$\{3 - t,\ 10 + 5\,t,\ 14 - 3\,t\}$$

```
solv1 = Flatten[Solve[ v[λ, μ] == w[t], {λ, μ , t}]]
```

$$\left\{\lambda \to \frac{18}{5},\ \mu \to -\frac{27}{5},\ t \to \frac{1}{5}\right\}$$

```
t0 = t /. solv1
```

$$\frac{1}{5}$$

```
w[t0]
```

$$\left\{\frac{14}{5},\ 11,\ \frac{67}{5}\right\}$$

```
N[%]
```

```
{2.8, 11., 13.4}
```

4

```
Remove["Global`*"]
```

```
p1 = {1, 1, 0}; p2 = {-1, 2, 2}; p3 = {-3, -2, 3}; p4 = {1, 1, 4};
m = {v1 = p2 - p1, v2 = p3 - p2, v3 = p4 - p3}
```

$$\{\{-2, 1, 2\}, \{-2, -4, 1\}, \{4, 3, 1\}\}$$

```
m + 1 / 2 m
```

$$\left\{\left\{-3, \frac{3}{2}, 3\right\}, \left\{-3, -6, \frac{3}{2}\right\}, \left\{6, \frac{9}{2}, \frac{3}{2}\right\}\right\}$$

```
s = Sum[1 / 2^k, {k, 0, 99}]
```

$$\frac{1267650600228229401496703205375}{633825300114114700748351602688}$$

```
s // N
```

$$2.$$

```
s m
```

$$\left\{\left\{-\frac{1267650600228229401496703205375}{316912650057057350374175801344}, \frac{1267650600228229401496703205375}{633825300114114700748351602688}, \frac{1267650600228229401496703205375}{316912650057057350374175801344}\right\}, \left\{-\frac{1267650600228229401496703205375}{316912650057057350374175801344}, -\frac{1267650600228229401496703205375}{158456325028528675187087900672}, \frac{1267650600228229401496703205375}{633825300114114700748351602688}\right\}, \left\{\frac{1267650600228229401496703205375}{158456325028528675187087900672}, \frac{380295180068468820449010961612 5}{633825300114114700748351602688}, \frac{1267650600228229401496703205375}{633825300114114700748351602688}\right\}\right\}$$

```
s m // N
```

$$\{\{-4., 2., 4.\}, \{-4., -8., 2.\}, \{8., 6., 2.\}\}$$

```
TotalPunkte =
 (3 * 100 + 1) Punkte (* je 3 Anfangspunkte von λ^0 bis λ^99 plus Endpunkt *)
```

$$301\ \text{Punkte}$$

```
p301 = p1 + Apply[Plus, s m] // N
```

$$\{1., 1., 8.\}$$

5

```
Remove["Global`*"]
```

```
Solve[{x + y + z == 1, x - y + z == 1, x - y - z == 0}, {x, y, z}]
```

$$\left\{\left\{x \to \frac{1}{2}, y \to 0, z \to \frac{1}{2}\right\}\right\}$$

6

```
Remove["Global`*"]
```

```
A = {{1, 2, 3}, {2, 3, 1}, {0, 1, 1}}; A // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

```
B = {{1, 0, 3}, {2, 3, 1}, {0, 1, 1}}; A // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

```
A.B // MatrixForm
```

$$\begin{pmatrix} 5 & 9 & 8 \\ 8 & 10 & 10 \\ 2 & 4 & 2 \end{pmatrix}$$

```
Inverse[A] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & -\frac{7}{4} \\ -\frac{1}{2} & \frac{1}{4} & \frac{5}{4} \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

```
Inverse[A] // N // MatrixForm
```

$$\begin{pmatrix} 0.5 & 0.25 & -1.75 \\ -0.5 & 0.25 & 1.25 \\ 0.5 & -0.25 & -0.25 \end{pmatrix}$$

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4} & \frac{3}{8} & -\frac{9}{8} \\ -\frac{1}{4} & \frac{1}{8} & \frac{5}{8} \\ \frac{1}{4} & -\frac{1}{8} & \frac{3}{8} \end{pmatrix}$$

```
Inverse[B] // N // MatrixForm
```

$$\begin{pmatrix} 0.25 & 0.375 & -1.125 \\ -0.25 & 0.125 & 0.625 \\ 0.25 & -0.125 & 0.375 \end{pmatrix}$$

```
Transpose[Inverse[A.B]] // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{7}{16} & -\frac{3}{16} & -\frac{1}{16} \\ \frac{5}{16} & \frac{7}{16} & -\frac{11}{16} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.625 & 0.125 & 0.375 \\ 0.4375 & -0.1875 & -0.0625 \\ 0.3125 & 0.4375 & -0.6875 \end{pmatrix}$$

```
Transpose[Inverse[B].Inverse[A]] == Transpose[Inverse[A.B]]
```

```
True
```

7

```
M = {{1, -1}, {2, 1}}; p0 = {4, 7}; M // MatrixForm
```

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

```
p1 = M.p0
```

```
{-3, 15}
```

```
α = 32 Degree; d32 = {{Cos[α], -Sin[α]}, {Sin[α], Cos[α]}}; d32 // N // MatrixForm
```

$$\begin{pmatrix} 0.848048 & -0.529919 \\ 0.529919 & 0.848048 \end{pmatrix}$$

```
p2 = d32.p1
```

```
{-3 Cos[32 °] - 15 Sin[32 °], 15 Cos[32 °] - 3 Sin[32 °]}
```

```
p2 // N
```

```
{-10.4929, 11.131}
```

```
p3 = M.p2
```

```
{-18 Cos[32 °] - 12 Sin[32 °], 15 Cos[32 °] + 2 (-3 Cos[32 °] - 15 Sin[32 °]) - 3 Sin[32 °]}
```

```
p3 // N
```

```
{-21.6239, -9.8549}
```

8

```
Remove["Global`*"]
```

```
p1 = {1, 1, 0}; p2 = {1, 0, 2}; p3 = {0, 2, 3};
```

```
S = 1 / 3 (p1 + p2 + p3)
```

$$\left\{\frac{2}{3}, 1, \frac{5}{3}\right\}$$

```
N[%]
```

```
{0.666667, 1., 1.66667}
```

```
Vp1p2p3 = 1 / 2 Norm[Cross[p2 - p1, p3 - p1]]
```

$$\sqrt{\frac{15}{2}}$$

```
N[%]
```

```
2.73861
```

```
p[t_] := t S;
```

```
norm[v_] := Sqrt[v.v]
```

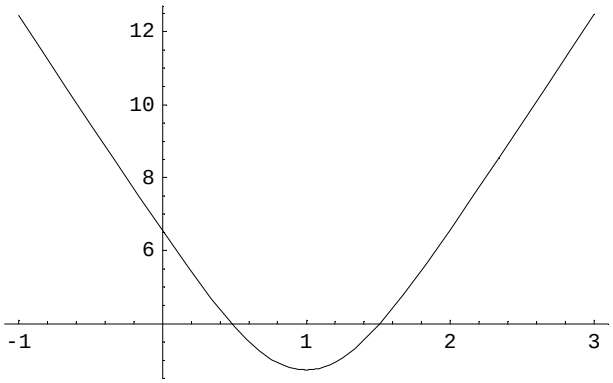
```
Vof[t_] := 1 / 2 (norm[Cross[p2 - p1, p[t] - p1]] + norm[Cross[p3 - p2, p[t] - p2]] +
    norm[Cross[p1 - p3, p[t] - p3]]);  Vof[t] // ExpandAll
```

$$\frac{1}{2}\sqrt{9-\frac{64\,t}{3}+\frac{47\,t^2}{3}}+\frac{1}{2}\sqrt{29-42\,t+\frac{49\,t^2}{3}}+\frac{1}{2}\sqrt{22-\frac{110\,t}{3}+18\,t^2}$$

```
solv = Solve[2 Vp1p2p3 == Vof[t], {t}] // N // Flatten
```

```
{t → 0.1902, t → 1.80319}
```

```
Plot[Vof[t], {t, -1, 3}]
```




```
- Graphics -
```

```
p[t] /. solv[[2]]
```

```
{1.20213, 1.80319, 3.00531}
```

9

```
Remove["Global`*"]
```

```
r1 = {1, 2, -1};  r2 = {-1, 1, 3};  a = {2, 1, 1};  b = {3, -1, 2};
```

```
v1[t_] := r1 + t a;  v2[r_] := r2 + r b;
```

a

```
V = Det[{a, b, r1 - r2}]
```

```
25
```

Determinante nicht 0, nicht windschief

b (Sprachproblem: was ist der negative oder der positive Fall?)

```
d = V / Norm[Cross[a, b]]
```

$$5\sqrt{\frac{5}{7}}$$

```
N[%]
```

```
4.22577
```

10

```
Remove["Global`*"]
```

a, b

```
r = 2; k[x_, y_, z_] := x^2 + y^2 + z^2 - r^2
```

```
ϕ[x_, y_, z_, a_] := x + y + z - a
```

```
ϕ[{x_, y_, z_}, a_] := ϕ[x, y, z, a]
```

```
k[x, x, x] == 0
```

$$-4 + 3\,x^2 == 0$$

```
solv = Solve[ k[x, x, x] == 0, {x}] // Flatten
```

$$\left\{x \to -\frac{2}{\sqrt{3}},\ x \to \frac{2}{\sqrt{3}}\right\}$$

```
TPunkt = {2/Sqrt[3], 2/Sqrt[3], 2/Sqrt[3]}
```

$$\left\{\frac{2}{\sqrt{3}},\ \frac{2}{\sqrt{3}},\ \frac{2}{\sqrt{3}}\right\}$$

```
N[%]
```

```
{1.1547, 1.1547, 1.1547}
```

```
ϕ[TPunkt, a] == 0
```

$$2\sqrt{3} - a == 0$$

```
Solve[ϕ[TPunkt, a] == 0, {a}]
```

$$\{\{a \to 2\sqrt{3}\}\}$$

c

```
e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1}; T = TPunkt;
```

```
α = ArcCos[T.e1 / Norm[T] / Norm[e1]]
```

$$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$$

```
β = ArcCos[T.e2 / Norm[T] / Norm[e2]]
```

$$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$$

```
γ = ArcCos[T.e3 / Norm[T] / Norm[e3]]
```

$$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$$

```
N[%]
```

```
0.955317
```

```
% / Degree
```

```
54.7356
```