Lösungen

1

```
Remove["Global`*"]
```

```
N[563 / 7, 35]
```

```
80.428571428571428571428571428571429
```

```
Rationalize[80.428571428571428571428571428571428571428571428572]
```

$$\frac{563}{7}$$

```
a1 = 80.428571428571428571428571428571428571428571428572
```

```
80.428571428571428571428571428571428571428571428572
```

```
10^6 a1
```

$$8.042857142857142857142857142857142857142857142857 \times 10^{7}$$

```
Round[10^6 a1 - a1]
```

```
80428491
```

```
Round[10^6 a1 - a1] / (10^6 - 1)
```

$$\frac{563}{7}$$

```
80428491 / 999999
```

$$\frac{563}{7}$$

```
8936499 / 111111
```

$$\frac{563}{7}$$

2

```
Remove["Global`*"]
```

Zur Lösung

```
Solve[{Abs[2 λ x] + w == 0, 2 x + y == 14, 3 x + z == 56, z - w == 42}, {x, y, z, w}] // Simplify
```

$$\left\{\left\{y \to \frac{14 - 28\,\lambda}{3 - 2\,\lambda},\ z \to \frac{14\,(-9 + 8\,\lambda)}{-3 + 2\,\lambda},\ x \to \frac{14}{3 - 2\,\lambda},\ w \to \frac{28\,\lambda}{-3 + 2\,\lambda}\right\},\right.$$
$$\left.\left\{y \to \frac{14 + 28\,\lambda}{3 + 2\,\lambda},\ z \to \frac{14\,(9 + 8\,\lambda)}{3 + 2\,\lambda},\ x \to \frac{14}{3 + 2\,\lambda},\ w \to \frac{28\,\lambda}{3 + 2\,\lambda}\right\}\right\}$$

Sei SignX der Ausdruck SignX = Abs[ ]*Sign[x]. Es gilt damit: SignX*x = Abs[ ]*Sign[x]*x = Abs[*x]. Das Vorzeichen von SignX ist daher identisch mit dem Vorzeichen von x. x existiert also nur, wenn x>0 und SignX > 0 ist, was für 0 < < 3/2 eintrifft. In allen anderen Fällen sind die Vorzeichen von x und Sign unterschiedlich.

In[170]:=

```
Solve[{2 μSignX x + w == 0, 2 x - y == 14, 3 x + z == 56, z - w == 42}, {x, y, z, w}] // Simplify
```

Out[170]=

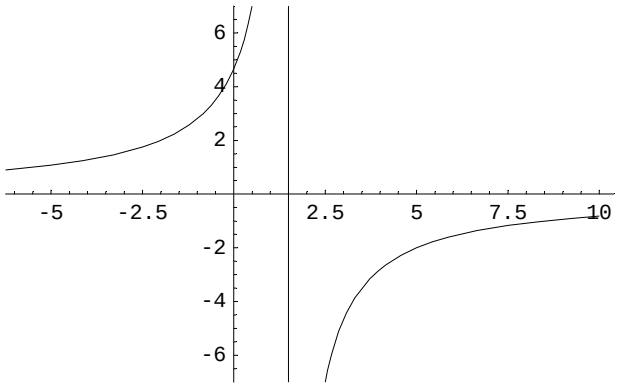
$$\left\{\left\{y \to \frac{14 - 28\,\mu\text{SignX}}{-3 + 2\,\mu\text{SignX}},\ z \to \frac{14\,(-9 + 8\,\mu\text{SignX})}{-3 + 2\,\mu\text{SignX}},\ x \to \frac{14}{3 - 2\,\mu\text{SignX}},\ w \to \frac{28\,\mu\text{SignX}}{-3 + 2\,\mu\text{SignX}}\right\}\right\}$$

In[172]:=

```
x1[μSignX_] := 14 / (3 - 2 μSignX)
```

In[174]:=

```
Plot[x1[μSignX], {μSignX, -10, 10}, PlotRange → {-7, 7}];
```



Weitere Betrachtungen

```
Solve[{2 Abs[λ] Sgnx x + w == 0, 2 x - y == 14, 3 x + z == 56, z - w == 42}, {x, y, z, w}] //
 Simplify
```

$$\left\{\left\{y \to -14 + \frac{28}{3 - 2\,\text{Sgnx Abs}[\lambda]},\ z \to \frac{14\,(-9 + 8\,\text{Sgnx Abs}[\lambda])}{-3 + 2\,\text{Sgnx Abs}[\lambda]},\right.\right.$$
$$\left.\left.x \to \frac{14}{3 - 2\,\text{Sgnx Abs}[\lambda]},\ w \to \frac{28\,\text{Sgnx Abs}[\lambda]}{-3 + 2\,\text{Sgnx Abs}[\lambda]}\right\}\right\}$$

```
Solve[{2 Abs[λ] Sign[x] x + w == 0, 2 x - y == 14, 3 x + z == 56, z - w == 42}, {x, y, z, w}] //
 Simplify
```

$$\left\{\left\{y \to \frac{14 - 28\,\text{Abs}[\lambda]}{-3 + 2\,\text{Abs}[\lambda]},\ z \to \frac{14\,(-9 + 8\,\text{Abs}[\lambda])}{-3 + 2\,\text{Abs}[\lambda]},\ w \to \frac{28\,\text{Abs}[\lambda]}{-3 + 2\,\text{Abs}[\lambda]},\ x \to \frac{14}{3 - 2\,\text{Abs}[\lambda]}\right\},\right.$$
$$\left.\left\{y \to -\frac{14\,(1 + 2\,\text{Abs}[\lambda])}{3 + 2\,\text{Abs}[\lambda]},\ z \to \frac{14\,(9 + 8\,\text{Abs}[\lambda])}{3 + 2\,\text{Abs}[\lambda]},\ w \to \frac{28\,\text{Abs}[\lambda]}{3 + 2\,\text{Abs}[\lambda]},\ x \to \frac{14}{3 + 2\,\text{Abs}[\lambda]}\right\}\right\}$$

```
Solve[{2 Abs[λ] Sign[x] x + w == 0, 2 x - y == 14, 3 x + z == 56, z - w == 42}, {λ, y, z, w}] //
 Simplify
```

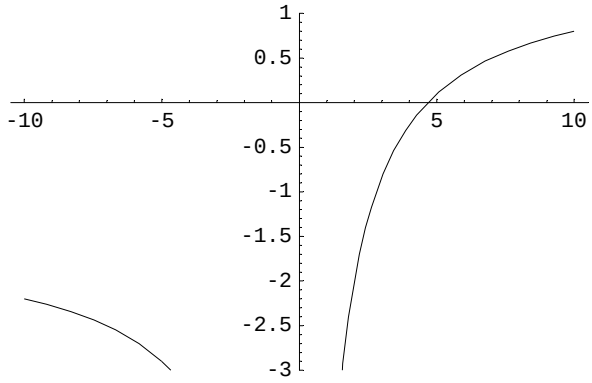
$$\left\{\left\{\lambda \to \frac{-14+3x}{2x\,\text{Sign}[x]},\ y \to 2(-7+x),\ w \to 14-3x,\ z \to 56-3x\right\},\right.$$
$$\left.\left\{\lambda \to \frac{14-3x}{2x\,\text{Sign}[x]},\ y \to 2(-7+x),\ w \to 14-3x,\ z \to 56-3x\right\}\right\}$$

```
Solve[{2 μ Sign[x] x + w == 0, 2 x - y == 14, 3 x + z == 56, z - w == 42}, {μ, y, z, w}] // Simplify
```

$$\left\{\left\{\mu \to \frac{-14+3x}{2x\,\text{Sign}[x]},\ y \to 2(-7+x),\ w \to 14-3x,\ z \to 56-3x\right\}\right\}$$

```
μ[x_] := (-14 + 3 x)/(2 x Sign[x])
```

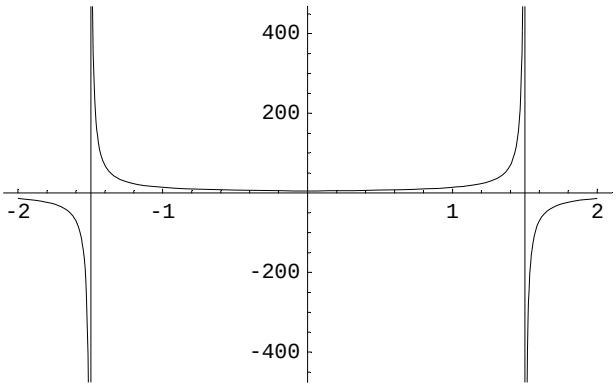
```
Plot[μ[u], {u, -10, 10}, PlotRange → {-3, 1}];
```



Nur der positive Bereich von brauchbar.

```
x[λ_] := 14/(3 - 2 Abs[λ])
```

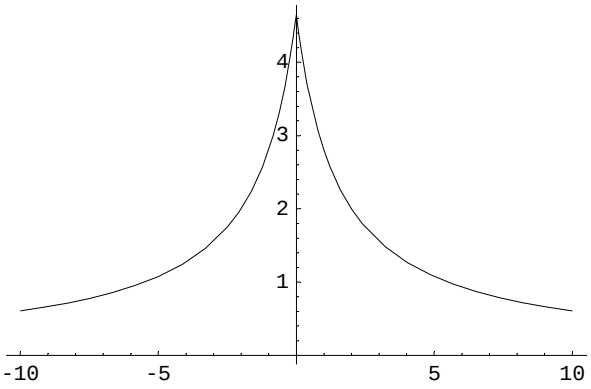
```
Plot[x[u], {u, -2, 2}];
```



Wenn = Abs[ ] Sign[x] und somit auch x positiv ist, sind die positiven x-Werte Lösung. Das kommt hier vor, wenn zwischen 0 und 3/2 = 1.5 liegt.

```
x[λ_] := 14/(3 - (-2 Abs[λ]))
```

```
Plot[x[u], {u, -10, 10}];
```



Wenn = Abs[ ] Sign[x] und somit auch x negativ ist, sind die negativen x-Werte Lösung. Das kommt hier nicht vor.

3

```
Remove["Global`*"]
```

```
4 / Log[x] Log[x, x / x^Log[2, x] a^Log[2, x]] -
 8 Log[2, 4^Log[2, a]] Log[2, 16] / Log[2, a] + 12 Log[2, Log[4, 16]^Log[2, 8]]
```

$$36 - \frac{32 \operatorname{Log}\left[4^{\frac{\operatorname{Log}[a]}{\operatorname{Log}[2]}}\right]}{\operatorname{Log}[a]} + \frac{4 \operatorname{Log}\left[a^{\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\, x^{1-\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\right]}{\operatorname{Log}[x]^2}$$

```
4 / Log[x] Log[x, x / x^Log[2, x] a^Log[2, x]] -
  8 Log[2, 4^Log[2, a]] Log[2, 16] / Log[2, a] +
  12 Log[2, Log[4, 16]^Log[2, 8]] /. Log[s__^t__] → t Log[s]
```

$$36 - \frac{32 \operatorname{Log}[4]}{\operatorname{Log}[2]} + \frac{4 \operatorname{Log}\left[a^{\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\, x^{1-\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\right]}{\operatorname{Log}[x]^2}$$

```
(36 - 32 Log[s^t] / Log[2]) /. Log[s_^t_] → t Log[s]
```

$$36 - \frac{32\, t \operatorname{Log}[s]}{\operatorname{Log}[2]}$$

```
36 - 32 t Log[s] / Log[2] /. {s → 2, t → 2}
```

$$-28$$

```
4 Log[a^(Log[x]/Log[2]) x^(1 - Log[x]/Log[2])] / Log[x]^2 /. Log[s__ * t__] → Log[s] + Log[t]
```

$$\frac{4 \left(\operatorname{Log}\left[a^{\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\right] + \operatorname{Log}\left[x^{1-\frac{\operatorname{Log}[x]}{\operatorname{Log}[2]}}\right]\right)}{\operatorname{Log}[x]^2}$$

```
4 (Log[a^(Log[x]/Log[2])] + Log[x^(1 - Log[x]/Log[2])])/Log[x]^2 /. Log[s_^t_] → t Log[s]
```

$$\frac{4\left(\frac{\text{Log}[a]\,\text{Log}[x]}{\text{Log}[2]}+\text{Log}[x]\left(1-\frac{\text{Log}[x]}{\text{Log}[2]}\right)\right)}{\text{Log}[x]^2}$$

```
4 (Log[a] Log[x]/Log[2] + Log[x] (1 - Log[x]/Log[2]))/Log[x]^2 // Simplify
```

$$\frac{4\,(\text{Log}[2]+\text{Log}[a]-\text{Log}[x])}{\text{Log}[2]\,\text{Log}[x]}$$

```
-28 + 4 (Log[2] + Log[a] - Log[x])/(Log[2] Log[x]) // Simplify
```

$$-28+\frac{4\,(\text{Log}[2]+\text{Log}[a]-\text{Log}[x])}{\text{Log}[2]\,\text{Log}[x]}$$

4

```
Remove["Global`*"]
```

```
Solve[Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])] == 0, {y}]
```

$$\{\{y\to 10000000000\},\{y\to 10^{1/10}\}\}$$

```
10000000000 // N
```

$$1.\times 10^{10}$$

```
10^(1/10) // N
```

$$1.25893$$

```
Solve[Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])] == 0, {y}] // N
```

$$\{\{y\to 1.\times 10^{10}\},\{y\to 1.25893\}\}$$

```
Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])] /. y → (10^(1 / 10))
```

$$0$$

```
Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])] /.  Log[x_] → log[x]
```

$$\sqrt{\left(1-\frac{\log\left[\frac{\text{Log}[y]}{\text{Log}[10]}\right]}{\log[10]}\right)\left(1+\frac{\log\left[\frac{\text{Log}[y]}{\text{Log}[10]}\right]}{\log[10]}\right)}$$

```
Sqrt[(1 - log[Log[y]/Log[10]]/log[10]) (1 + log[Log[y]/Log[10]]/log[10])] /. Log[x_] → log[x]
```

$$\sqrt{\left(1-\frac{\log\left[\frac{\log[y]}{\log[10]}\right]}{\log[10]}\right)\left(1+\frac{\log\left[\frac{\log[y]}{\log[10]}\right]}{\log[10]}\right)}$$

```
(Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])] /. y → (10^10)) // N // Chop
```

2.10734×10^{-8}

Sollte 0 geben, wird aber bei Vorauseinstellung der Genauigkeit nicht exakt.

```
N[(Evaluate[Sqrt[(1 - Log[10, Log[10, y]]) (1 + Log[10, Log[10, y]])]] /. y → (10^10)),
  30] // Chop
```

0

5

```
Remove["Global`*"]
```

```
Solve[Log[u]^2 + Log[Sqrt[u]] - 9 == Log[E u], {u}]
```

$\text{Solve}\left[-9 + \text{Log}\left[\sqrt{u}\right] + \text{Log}[u]^2 == \text{Log}[e\, u], \{u\}\right]$

```
(Log[u]^2 + Log[u^(1 / 2)] - 9 == Log[E u]) /.
 {Log[s_^t_] → t Log[s], Log[s_ t_] → Log[s] + Log[t]}
```

$-9 + \frac{\text{Log}[u]}{2} + \text{Log}[u]^2 == 1 + \text{Log}[u]$

```
-9 + Log[u]/2 + Log[u]^2 == 1 + Log[u] /. Log[u] → v
```

$-9 + \frac{v}{2} + v^2 == 1 + v$

```
solv = (Solve[-9 + v/2 + v^2 == 1 + v, {v}] // Flatten)
```

$\left\{v \to \frac{1}{4}\left(1 - \sqrt{161}\right),\ v \to \frac{1}{4}\left(1 + \sqrt{161}\right)\right\}$

```
v1 = v /. solv[[1]]
```

$\frac{1}{4}\left(1 - \sqrt{161}\right)$

```
v2 = v /. solv[[2]]
```

$\frac{1}{4}\left(1 + \sqrt{161}\right)$

```
Solve[Log[u] == v1, {u}]
```

$\left\{\left\{u \to e^{\frac{1}{4} - \frac{\sqrt{161}}{4}}\right\}\right\}$

```
Solve[Log[u] == v1, {u}] // N
```

$\{\{u \to 0.0538182\}\}$

```
Solve[Log[u] == v2, {u}]
```

$\left\{\left\{u \to e^{\frac{1}{4} + \frac{\sqrt{161}}{4}}\right\}\right\}$

```
Solve[Log[u] == v2, {u}] // N
```

```
{{u → 30.635}}
```

```
((Log[u]^2 + Log[Sqrt[u]] - 9 == Log[E u]) // N) /. u -> e^(1/4 - Sqrt[161]/4)
```

```
True
```

```
((Log[u]^2 + Log[Sqrt[u]] - 9 == Log[E u]) // N) /. u -> e^(1/4 + Sqrt[161]/4)
```

```
True
```

6

```
Remove["Global`*"]
```

```
Sin[x 17^(1 - 5^0)]^2 + E^(Sin[3 Pi]) == 11011 / 100000 - Cos[x]^2
```

$$1 + \text{Sin}[x]^2 == \frac{11011}{100000} - \text{Cos}[x]^2$$

```
Solve[Sin[x 17^(1 - 5^0)]^2 + E^(Sin[3 Pi]) == 11011 / 100000 - Cos[x]^2, {x}]
```

```
{}
```

7

```
Remove["Global`*"]
```

```
4^(2 (x - 1)) 6^(3 x) 8^(-1) // Simplify
```

$$27^x\, 128^{-1+x}$$

```
3^(4 - 2 x) 4^(3 - 3 x) // Simplify
```

$$4^{3-3x}\, 9^{2-x}$$

```
27^x 128^x (4^3)^x 9^x == 4^3 9^2 128
```

$$1990656^x == 663552$$

```
(4^(2 (x - 1)) 6^(3 x) 8^(-1) == 3^(4 - 2 x) 4^(3 - 3 x)) // Simplify // ExpandAll
```

$$-2^{13-x}\, 9^{2-x} + 110592^x == 0$$

```
(27 * 128 * 4^3 * 9)^x == 128^1 * 4^3 * 9^2
```

$$1990656^x == 663552$$

```
Solve[1990656^x == 663552, {x}]
```

$$\left\{\left\{x \to \frac{\text{Log}[663552]}{\text{Log}[1990656]}\right\}\right\}$$

```
{{x → Log[663552]/Log[1990656]}} // Flatten // Simplify
```

$$\left\{x \to \frac{\text{Log}[663552]}{\text{Log}[1990656]}\right\}$$

```
Solve[1990656^x == 663552, {x}] // N
```

$$\{\{x \to 0.924254\}\}$$

```
(4 Log[3] + 13 Log[2]) / (5 Log[3] + 13 Log[2]) // N
```

```
0.924254
```

8

```
Remove["Global`*"]
```

```
Solve[{y == (x - 2) (x + 3), y == x / 2 + u}, {x, y}]
```

$$\left\{\left\{y \to \frac{1}{8}\left(-1 + 8\,u - \sqrt{97 + 16\,u}\right),\ x \to \frac{1}{4}\left(-1 - \sqrt{97 + 16\,u}\right)\right\},\right.$$
$$\left.\left\{y \to \frac{1}{2}\left(-\frac{1}{4} + 2\,u + \frac{1}{4}\sqrt{97 + 16\,u}\right),\ x \to \frac{1}{4}\left(-1 + \sqrt{97 + 16\,u}\right)\right\}\right\}$$

```
Solve[{y == (x - 2) (x + 3), y == x / 2 + u}, {x, y}] /. u → -97 / 16
```

$$\left\{\left\{y \to -\frac{99}{16},\ x \to -\frac{1}{4}\right\},\ \left\{y \to -\frac{99}{16},\ x \to -\frac{1}{4}\right\}\right\}$$

```
(Solve[{y == (x - 2) (x + 3), y == x / 2 + u}, {x, y}] /. u → -97 / 16) // N
```

$$\{\{y \to -6.1875,\ x \to -0.25\},\ \{y \to -6.1875,\ x \to -0.25\}\}$$

```
-97 / 16 // N
```

```
-6.0625
```

9

```
Remove["Global`*"]
```

```
z[x_, y_] := (y^(-2) - x^(-2))^(1 / 3) ((x - y)^9)^(1 / 3) x ;  z[x, y]
```

$$x\left(-\frac{1}{x^2} + \frac{1}{y^2}\right)^{1/3}\left((x - y)^9\right)^{1/3}$$

```
n[x_, y_] := ( (((x^2 - y^2)^(1 / 3) (y^(-1) - x^(-1)))^(1 / 2)) (x^3)^(1 / 2) (x - y)) ;
n[x, y]
```

$$\sqrt{x^3}\,(x - y)\sqrt{\left(-\frac{1}{x} + \frac{1}{y}\right)\left(x^2 - y^2\right)^{1/3}}$$

```
z[x, y] / n[x, y] // Simplify
```

$$\frac{x\left(-\frac{1}{x^2}+\frac{1}{y^2}\right)^{1/3}\left((x-y)^9\right)^{1/3}}{\sqrt{x^3}\,(x-y)\sqrt{\left(-\frac{1}{x}+\frac{1}{y}\right)(x^2-y^2)^{1/3}}}$$

```
(z[x, y] / n[x, y] /. {x → 3, y → 2}) // N
```

$$0.56007$$

```
z[x, y] /. (a_^b_)^c_ → a^(b c)
```

$$x\left(-\frac{1}{x^2}+\frac{1}{y^2}\right)^{1/3}(x-y)^3$$

```
n[x, y] /. (a_^b_)^c_ → a^(b c)
```

$$x^{3/2}\,(x-y)\sqrt{\left(-\frac{1}{x}+\frac{1}{y}\right)(x^2-y^2)^{1/3}}$$

```
u1 = z[x, y] / n[x, y] /. (a_^b_)^c_ → a^(b c)
```

$$\frac{\left(-\frac{1}{x^2}+\frac{1}{y^2}\right)^{1/3}(x-y)^2}{\sqrt{x}\,\sqrt{\left(-\frac{1}{x}+\frac{1}{y}\right)(x^2-y^2)^{1/3}}}$$

```
(1 / x + 1 / y) /. (1 / a_ + 1 / b_) → (a + b) / (a b)
```

$$\frac{x+y}{x\,y}$$

$$\frac{\left(\frac{x^2-y^2}{x^2\,y^2}\right)^{1/3}(x-y)^2}{\sqrt{x}\,\sqrt{\left(\frac{x-y}{x\,y}\right)(x^2-y^2)^{1/3}}}$$

$$\frac{(x-y)^2\left(\frac{x^2-y^2}{x^2\,y^2}\right)^{1/3}}{\sqrt{x}\,\sqrt{\frac{(x-y)\,(x^2-y^2)^{1/3}}{x\,y}}}$$

```
(x - y)^2 ((x^2 - y^2)/(x^2 y^2))^(1/3) /. (a_^b_)^c_ → a^(b c)
```

$$\frac{(x-y)^2\left(\frac{x^2-y^2}{x^2\,y^2}\right)^{1/3}}{\sqrt{x}\,\sqrt{\frac{(x-y)\,(x^2-y^2)^{1/3}}{x\,y}}}$$

```
(x - y)^2 ((x^2 - y^2)/(x^2 y^2))^(1/3) /. (a_^2 - b_^2) → (a - b) (a + b)
```

$$(x-y)^2\left(\frac{(x-y)\,(x+y)}{x^2\,y^2}\right)^{1/3}$$

```
u1 = (x - y)^7/3 (x + y) ^ (1 / 3) x^ (-2 / 3)  y^ (-2 / 3)
```

$$\frac{(x-y)^{7/3}\,(x+y)^{1/3}}{x^{2/3}\,y^{2/3}}$$

```
u2 = x^ (1 / 2) (x - y) ^ (1 / 2) (x - y) ^ (1 / 6) (x + y) ^ (1 / 6) x^ (-1 / 2)  y^ (-1 / 2)
```

$$\frac{(x-y)^{2/3}\,(x+y)^{1/6}}{\sqrt{y}}$$

```
u1 / u2
```

$$\frac{(x-y)^{5/3}\,(x+y)^{1/6}}{x^{2/3}\,y^{1/6}}$$

```
((x - y)^5/3 (x + y)^1/6 / x^2/3 y^1/6 /. {x → 3, y → 2}) // N
```

```
0.56007
```

```
(x - y)^9/6 (x^2 - y^2)^1/6 / x^2/3 y^1/6
```

$$\frac{(x-y)^{3/2}\,(x^2-y^2)^{1/6}}{x^{2/3}\,y^{1/6}}$$

```
((x - y)^9/6 (x^2 - y^2)^1/6 / x^2/3 y^1/6 /. {x → 3, y → 2}) // N
```

```
0.56007
```