Lösungen

1

```
Remove["Global`*"]
```

a

i

```
f[x_] := 1 / 5 x^5 + 1 / 4 x^4 + 1 / 3 x^3 + 1 / 2 x^2 + x - 1;
f[x]
```

$$-1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$$

```
f'[x]
```

$$1 + x + x^2 + x^3 + x^4$$

```
Evaluate[f'[x]] (x - 1) // Expand
```

$$-1 + x^5$$

ii

```
(Evaluate[f'[x]] // Expand) /. x → 1
```

```
5
```

iii

```
ArcTan[(Evaluate[f'[x]] // Expand) /. x → 1]
```

```
ArcTan[5]
```

```
N[%]
```

```
1.3734
```

b

i

```
f1[x_] := Log[3 x^2 + 1]; f1[x]
```

$$\text{Log}[1 + 3\,x^2]$$

```
f1'[x] // Simplify
```

$$\frac{6\,x}{1 + 3\,x^2}$$

```
r1 = (f1'[x] // Simplify) /. x → 0
```

$$0$$

```
f2[x_] := E^(a x); f2[x]
```

$$e^{a\,x}$$

```
f2'[x] // Simplify
```

$$a\,e^{a\,x}$$

```
r2 = (f2'[x] // Simplify) /. x → 0
```

$$a$$

```
f3[a_] := E^(a x); f3[a]
```

$$e^{a\,x}$$

```
f3'[a] // Simplify
```

$$e^{a\,x}\,x$$

```
r3 = (f3'[a] // Simplify) /. x → 0
```

$$0$$

```
f4[x_] := E^(2 a); f4[x]
```

$$e^{2\,a}$$

```
f4'[x] // Simplify
```

$$0$$

```
r4 = (f4'[x] // Simplify) /. x → 0
```

$$0$$

```
f5[x_] := Tan[x + 1] + Sin[x^2]; f5[x]
```

$$\text{Sin}[x^2] + \text{Tan}[1 + x]$$

```
f5'[x] // Simplify
```

$$2\,x\,\text{Cos}[x^2] + \text{Sec}[1+x]^2$$

```
r5 = (f5'[x] // Simplify) /. x → 0
```

$$\text{Sec}[1]^2$$

```
N[%]
```

$$3.42552$$

```
r1 + r2 + r3 + r4 + r5
```

$$a + \text{Sec}[1]^2$$

```
r1 + r2 + r3 + r4 + r5 /. a → 1
```

$$1 + \text{Sec}[1]^2$$

```
N[%]
```

$$4.42552$$

ii

```
(f1'[x] // N) /. {x → Pi, a → 2}
```

$$0.615821$$

```
s[1] = ArcTan[(f1'[x] // N)] /. {x → Pi, a → 2}
```

$$0.551972$$

```
(f2'[x] // N) /. {x → Pi, a → 2}
```

$$1070.98$$

```
s[2] = ArcTan[(f2'[x] // N)] /. {x → Pi, a → 2}
```

$$1.56986$$

```
(f3'[x] // N) /. {x → Pi, a → 2}
```

$$60738.6$$

```
s[3] = ArcTan[(f3'[a] // N)] /. {x → Pi, a → 2}
```

$$1.5702$$

```
(f4'[x] // N) /. {x → Pi, a → 2}
```

$$0.$$

```
s[4] = ArcTan[(f4'[x] // N)] /. {x → Pi, a → 2}
```

$$0.$$

```
(f5'[x] // N) /. {x → Pi, a → 2}
```

$$-2.24622$$

```
s[5] = ArcTan[(f5'[x] // N)] /. {x → Pi, a → 2}
```

```
-1.15195
```

```
{Max[Table[s[k], {k, 1, 5}]], s[3]}
```

```
{1.5702, 1.5702}
```

```
a = 2
```

```
2
```

```
(f1'[x] // N) /. x → Pi
```

```
0.615821
```

```
s[3] = ArcTan[(f1'[x] // N)] /. x → Pi
```

```
0.551972
```

```
(f2'[x] // N) /. x → Pi
```

```
1070.98
```

```
s[2] = ArcTan[(f2'[x] // N)] /. x → Pi
```

```
1.56986
```

```
(f3'[x] // N) /. x → Pi
```

```
60738.6
```

```
s[3] = ArcTan[(f3'[a] // N)] /. x → Pi
```

```
1.5702
```

```
(f4'[x] // N) /. x → Pi
```

```
0.
```

```
s[4] = ArcTan[(f4'[x] // N)] /. x → Pi
```

```
0.
```

```
(f5'[x] // N) /. x → Pi
```

```
-2.24622
```

```
s[5] = ArcTan[(f5'[x] // N)] /. x → Pi
```

```
-1.15195
```

```
{Max[Table[s[k], {k, 1, 5}]], s[3]}
```

```
{1.5702, 1.5702}
```

c

```
g1[x_] := E^(2 x) / (x^2 - 1) + x Cos[x^2];
g1'[x] // Simplify
```

$$\text{Cos}[x^2] - \frac{2\left(e^{2x}\,(1+x-x^2)+x^2\,(-1+x^2)^2\,\text{Sin}[x^2]\right)}{(-1+x^2)^2}$$

```
(g1'[x] // Simplify) /. x → 0
```

$$-1$$

d

```
g2[x_] := -1 / ((x - 1) (x + 2));
g2'[x] // Simplify
```

$$\frac{1+2x}{(-2+x+x^2)^2}$$

```
(g2'[x] // Simplify) /. x → 0
```

$$\frac{1}{4}$$

2

```
Remove["Global`*"]
```

a

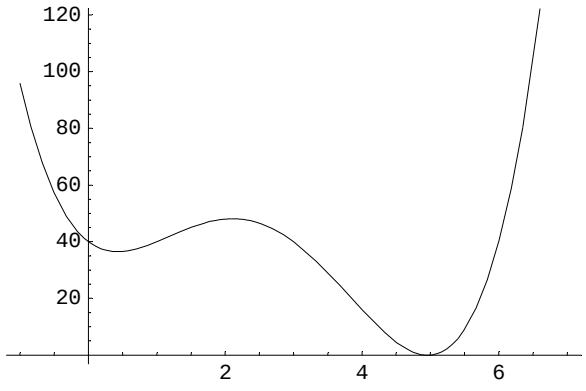
```
f[x_] := x (x - 1) (x - 3) (x - 6) + 40 // ExpandAll
```

```
f[x]
```

$$40 - 18x + 27x^2 - 10x^3 + x^4$$

b

```
Plot[f[x], {x, -1, 7}];
```



c

```
Evaluate[f'[x]] /. x → 0
```

```
-18
```

```
ArcTan[(Evaluate[f'[x]] /. x → 0)] / Degree // N
```

```
-86.8202
```

```
ArcTan[(Evaluate[f'[x]] /. x → 6)] / Degree // N
```

```
89.3634
```

d

```
solv = Solve[Evaluate[f'[x]] == 0, {x}] // N // Chop
```

```
{{x → 4.96217}, {x → 0.430295}, {x → 2.10753}}
```

```
Table[f[x] /. solv[[k]], {k, 1, 3}]
```

```
{-0.0375594, 36.4914, 48.1086}
```

e

```
solv1 = Solve[Evaluate[f''[x]] == 0, {x}] // N // Chop
```

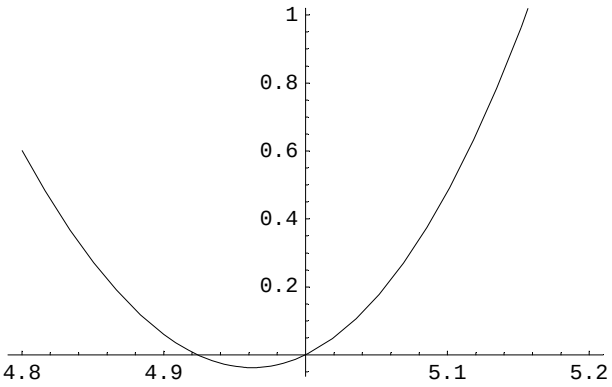
```
{{x → 1.17712}, {x → 3.82288}}
```

```
Table[f[x] /. solv1[[k]], {k, 1, 2}]
```

```
{41.833, 20.667}
```

f

```
Plot[f[x], {x, 4.8, 5.2}];
```




```
FindRoot[f[x] == 0, {x, 4.9}]
```

```
{x → 4.92379}
```

```
FindRoot[f[x] == 0, {x, 5.0}]
```

```
{x → 5.}
```

```
f[5]
```

```
0
```

3

```
Remove["Global`*"]
```

a

```
f[x_] := 1 / (x - 1)
```

```
Series[f[x], {x, 0, 1}]
```

```
-1 - x + O[x]^2
```

```
Series[f[x], {x, 0, 1}] // Normal
```

```
-1 - x
```

b

```
n3b = Normal[Series[f[x], {x, 0, 1}]] /. x → 0.1
```

```
-1.1
```

```
f3b = f[0.1]
```

```
-1.11111
```

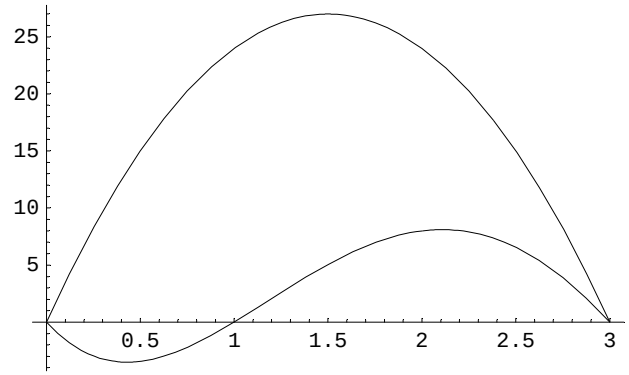
```
(n3b - f3b) / n3b * 100 // Abs
```

```
1.0101
```

4

```
Remove["Global`*"]
```

```
f[x_] := x (x - 1) (x - 3) (x - 6);
h[x_] := -12 x (x - 3);
Plot[{f[x], h[x]}, {x, 0, 3}];
```




```
Integrate[h[x] - f[x], {x, 0, 3}]
```

$$\frac{459}{10}$$

```
Integrate[h[x] - f[x], {x, 0, 3}] // N
```

```
45.9
```

5

```
Remove["Global`*"]
```

a

```
f[x_] := x Sin[x];
```

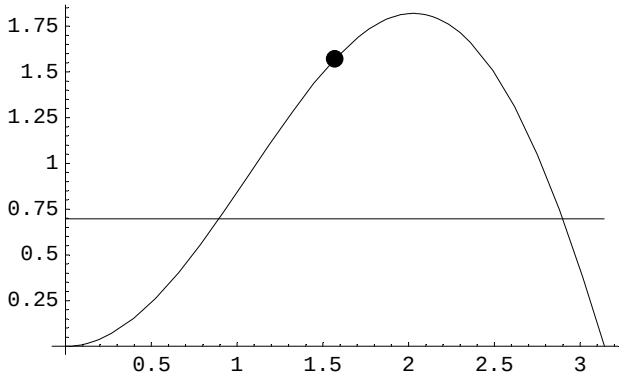
```
Integrate[f[x]^2 / 2, {x, 0, Pi}] / Integrate[f[x], {x, 0, Pi}]
```

$$\frac{1}{24}\left(-3+2\pi^2\right)$$

```
ky = N[%]
```

```
0.697467
```

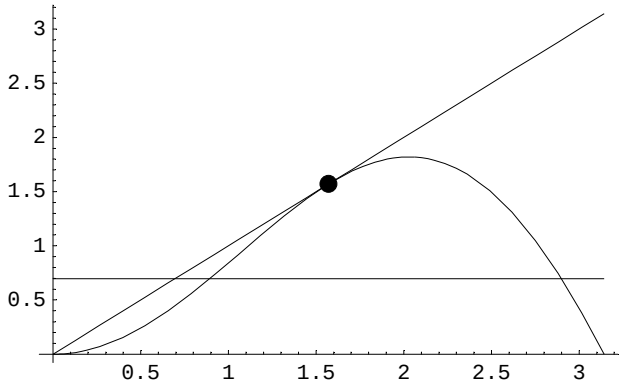
```
Plot[{ky, f[x]}, {x, 0, Pi}, Epilog → {PointSize[0.03], Point[{Pi / 2, f[Pi / 2]}]}];
```



b

```
g[x_] := Evaluate[f'[h] /. h → Pi / 2] (x - Pi / 2) + f[Pi / 2]
```

```
Plot[{ky, f[x], g[x]}, {x, 0, Pi},
  Epilog → {PointSize[0.03], Point[{Pi / 2, f[Pi / 2]}]}];
```




```
g[0]
```

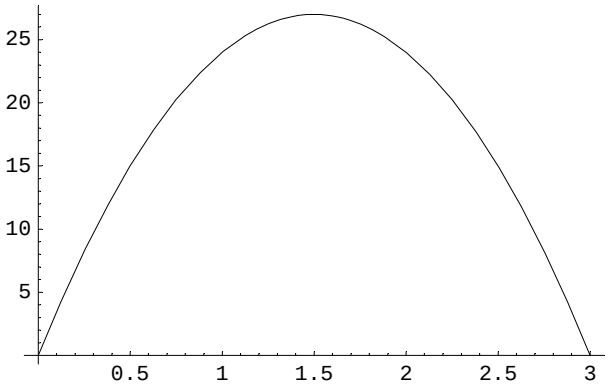
```
0
```

6

```
Remove["Global`*"]
```

```
h[x_] := -12 x (x - 3);
```

```
Plot[{h[x]}, {x, 0, 3}];
```



a

```
Laenge = Integrate[Evaluate[Sqrt[1 + (h'[x])^2]], {x, 0, 3}]
```

$$\frac{1}{24}\left(36\sqrt{1297}+\text{ArcSinh}[36]\right)$$

```
Laenge = Integrate[Evaluate[Sqrt[1 + (h'[x])^2]], {x, 0, 3}] // N
```

```
54.199
```

b

```
Inhalt = Pi Integrate[h[x]^2, {x, 0, 3}]
```

$$\frac{5832\,\pi}{5}$$

```
Inhalt = Pi Integrate[h[x]^2, {x, 0, 3}] // N
```

```
3664.35
```

c

```
Oberflaeche = 2 Pi Integrate[Evaluate[h[x] Sqrt[1 + (h'[x])^2]], {x, 0, 3}]
```

$$\frac{\pi\left(93276\sqrt{1297}+5185\,\text{ArcSinh}[36]\right)}{2304}$$

```
Oberflaeche = 2 Pi Integrate[Evaluate[h[x] Sqrt[1 + (h'[x])^2]], {x, 0, 3}] // N
```

```
4610.68
```

7

```
Remove["Global`*"]
```

```
DSolve[y'[x] == Cos[x] / y[x], y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\sqrt{2}\ \sqrt{C[1] + \text{Sin}[x]}]\}, \{y \to \text{Function}[\{x\}, \sqrt{2}\ \sqrt{C[1] + \text{Sin}[x]}]\}\}$$

```
DSolve[{y'[x] == Cos[x] / y[x], y[0] == 0}, y, x]
```

$$\{\{y \to \text{Function}[\{x\}, -\sqrt{2}\ \sqrt{\text{Sin}[x]}]\}, \{y \to \text{Function}[\{x\}, \sqrt{2}\ \sqrt{\text{Sin}[x]}]\}\}$$

```
Simplify[D[(Sqrt[2] Sqrt[Sin[x]]), x]] == Cos[x] / (Sqrt[2] Sqrt[Sin[x]])
```

```
True
```

8

```
Remove["Global`*"]
```

1+1/3+(1/3)^2+(1/3)^3+...

```
Sum[(1 / 3)^k, {k, 0, Infinity}]
```

$$\frac{3}{2}$$

```
1 / (1 - 1 / 3)
```

$$\frac{3}{2}$$

```
N[%]
```

```
1.5
```

(1)^(1/3)+(1/3)^(1/3)+(1/3)^(2/3)+(1/3)^(3/3)+... = ((1/3)^(1/3))^0+((1/3)^(1/3))^1+((1/3)^(1/3))^2+((1/3)^(1/3))^3+...

```
Sum[((1 / 3)^(1 / 3))^k, {k, 0, Infinity}]
```

$$-\frac{3}{-3 + 3^{2/3}}$$

```
N[%]
```

```
3.26117
```

```
1 / (1 - (1 / 3)^(1 / 3))
```

$$\frac{1}{1 - \frac{1}{3^{1/3}}}$$

```
N[%]
```

```
3.26117
```

9

```
Remove["Global`*"]
```

```
Solve[4000 == r^2 Pi h + 4 / 3 / 2 r^3 Pi, {h}]
```

$$\left\{\left\{h \to -\frac{2\,(-6000+\pi\,r^3)}{3\,\pi\,r^2}\right\}\right\}$$

```
Oberfl[r_] := 4 / 2 r^2 Pi + r^2 Pi + 2 r Pi h /. h → -(2 (-6000 + π r^3)) / (3 π r^2)
```

```
Oberfl[r]
```

$$3\,\pi\,r^2 - \frac{4\,(-6000+\pi\,r^3)}{3\,r}$$

```
Oberfl'[r]
```

$$2\,\pi\,r + \frac{4\,(-6000+\pi\,r^3)}{3\,r^2}$$

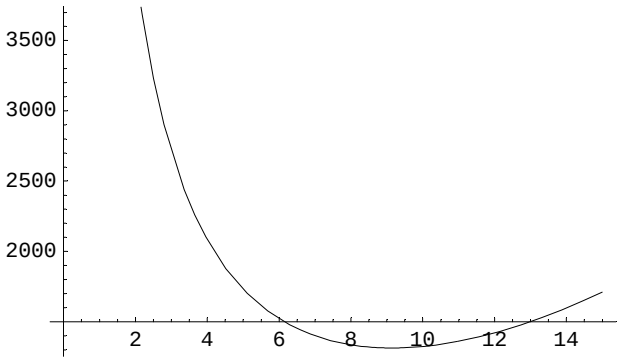
```
Solve[Evaluate[Oberfl'[r] == 0], {r}]
```

$$\left\{\left\{r \to -2\,10^{2/3}\left(-\frac{3}{\pi}\right)^{1/3}\right\},\ \left\{r \to 2\,(-10)^{2/3}\left(\frac{3}{\pi}\right)^{1/3}\right\},\ \left\{r \to 2\,10^{2/3}\left(\frac{3}{\pi}\right)^{1/3}\right\}\right\}$$

```
N[%]
```

$$\{\{r \to -4.57078 - 7.91683\,i\},\ \{r \to -4.57078 + 7.91683\,i\},\ \{r \to 9.14156\}\}$$

```
Plot[Oberfl[r], {r, 1, 15}];
```



10

```
Remove["Global`*"]
```

```
EmissA = 2 EmissB; rA = 5 - rB;
EmissA / rA^2 + EmissB / rB^2
```

$$\frac{2\,\text{EmissB}}{(5-\text{rB})^2} + \frac{\text{EmissB}}{\text{rB}^2}$$

```
f[rB_] := 2 EmissB/(5 - rB)^2 + EmissB/rB^2;
f'[rB] == 0
```

$$\frac{4\,\text{EmissB}}{(5-\text{rB})^3} - \frac{2\,\text{EmissB}}{\text{rB}^3} == 0$$

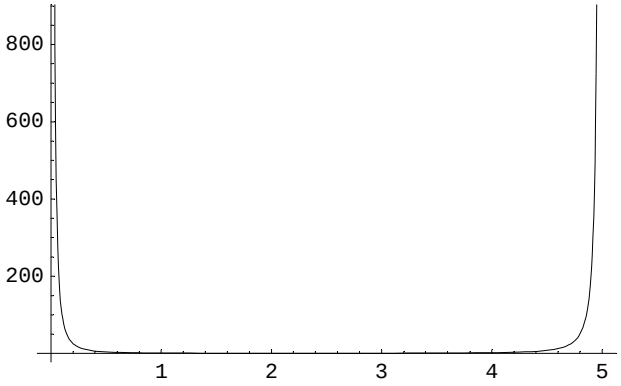
```
Solve[Evaluate[f'[rB] == 0], {rB}]
```

$$\left\{\left\{\text{rB} \to \frac{5}{3}\,(1 - 2^{1/3} + 2^{2/3})\right\}, \left\{\text{rB} \to \frac{5}{3} - \frac{5\left(1 - i\sqrt{3}\right)}{3\,2^{1/3}} + \frac{5\left(1 + i\sqrt{3}\right)}{3\,2^{2/3}}\right\},\right.$$
$$\left.\left\{\text{rB} \to \frac{5}{3} + \frac{5\left(1 - i\sqrt{3}\right)}{3\,2^{2/3}} - \frac{5\left(1 + i\sqrt{3}\right)}{3\,2^{1/3}}\right\}\right\}$$

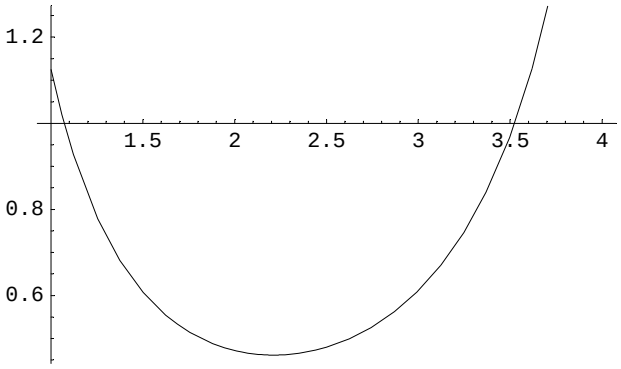
```
Solve[Evaluate[f'[rB] == 0], {rB}] // N
```

```
{{rB → 2.21247}, {rB → 1.39377 + 4.10976 i}, {rB → 1.39377 - 4.10976 i}}
```

```
Plot[(f[rB] /. EmissB → 1), {rB, 0, 5}];
```




```
Plot[(f[rB] /. EmissB → 1), {rB, 1, 4}];
```



Von B aus: 2.2124666701222107`

Von A aus:

```
5 - 2.2124666701222107`
```

```
2.78753
```