Lösungen

1

$$(A * X * A^{-1})\,(A * X * A^{-1}) + A^{T} * X * A^{-1} = M - A^{-1} + M^{T} + A * X^{2} * A^{-1}$$

$$A * X * X * A^{-1} + (A^{T} * X * A^{-1}) = M - A^{-1} + M^{T} + A * X^{2} * A^{-1}$$

$$A^{T} * X * A^{-1} = M - A^{-1} + M^{T}$$

$$A^{T} * X = M * A - A^{-1} * A + M^{T} * A$$

$$X = (A^{T})^{-1} * M * A - (A^{T})^{-1} + (A^{T})^{-1} * M^{T} * A$$

$$X = (A^{T})^{-1} * (M * A + M^{T} * A - E)$$

$$X = (A^{T})^{-1} * ((M + M^{T}) * A - E)$$

$$X = (A^{T})^{-1} * (A^{-1} * A^{T} * A - E)$$

2

```
Remove["Global`*"]

v1[t_] := {1, 2, -1} + λ {2, 1, 1};
v2[t_] := {-1, 1, 1} + μ {4, 5, -2};

V = Det[{{-1, 1, 1} - {1, 2, -1}, {2, 1, 1}, {4, 5, -2}}]
```

18

```
A = Norm[Cross[{2, 1, 1}, {4, 5, -2}]]
```

$\sqrt{149}$

```
d = (V / A)
```

$$\frac{18}{\sqrt{149}}$$

```
d = (V / A) // N
```

1.47462

3

```
Remove["Global`*"]

OA = {-1, 1, 1}; A = OA;

Φ[λ_, μ_] := OA + λ {2, 1, 1} + μ {4, 5, -2};
Q = {10, -10, 12};
```

a

```
V = Det[{{2, 1, 1}, {4, 5, -2}, Q - {-1, 1, 1}}]
```

$$-99$$

```
V / 6
```

$$-\frac{33}{2}$$

```
N[%]
```

$$-16.5$$

b

```
d = V / Norm[Cross[{2, 1, 1}, {4, 5, -2}]]
```

$$-\frac{99}{\sqrt{149}}$$

```
-d // N
```

$$8.1104$$

c

```
n = Cross[{2, 1, 1}, {4, 5, -2}]
```

$$\{-7, 8, 6\}$$

```
g[t_] := Q + t n;
solv = Solve[Φ[λ, μ] == g[t], {λ, μ, t}] // Flatten
```

$$\left\{\lambda \to \frac{1353}{149}, \mu \to -\frac{440}{149}, t \to \frac{99}{149}\right\}$$

```
L = g[t] /. solv
```

$$\left\{\frac{797}{149}, -\frac{698}{149}, \frac{2382}{149}\right\}$$

```
N[%]
```

```
{5.34899, -4.68456, 15.9866}
```

d

```
Q1 = Q + 2 (L - Q)
```

$$\left\{\frac{104}{149}, \frac{94}{149}, \frac{2976}{149}\right\}$$

```
N[%]
```

```
{0.697987, 0.630872, 19.9732}
```

e

```
ArcCos[(Q - A).(Q1 - A) / (Norm[Q - A] Norm[Q1 - A])]
```

$$\text{ArcCos}\left[\frac{95}{149}\right]$$

```
N[%]
```

```
0.879438
```

```
% / Degree
```

```
50.3881
```

4

```
Remove["Global`*"]
```

```
OA = {-1, 1, 1}; A = OA;
```

```
Φ1[x_, y_, z_] := (3 x + 5 y - 5 z - 12 == 0);
Φ2[x_, y_, z_] := (-4 x + 5 y + 3 z + 12 == 0);
OQ = {1, 2, 10};
```

```
solv4 = Solve[{Φ1[x, y, z], Φ2[x, y, z]}, {x, y, z}] // Flatten
```

$$\left\{x \to \frac{24}{7} + \frac{8\,z}{7},\ y \to \frac{12}{35} + \frac{11\,z}{35}\right\}$$

```
p[u_] := ({x, y, z} /. solv4) /. z → u;  p[u]
```

$$\left\{\frac{24}{7} + \frac{8\,u}{7}, \frac{12}{35} + \frac{11\,u}{35}, u\right\}$$

```
n = 35 (p[1] - p[0])
```

```
{40, 11, 35}
```

```
n1 = {3, 5, -5}; n2 = {-4, 5, 3};
nn = Cross[n1, n2]
```

```
{40, 11, 35}
```

```
solv4a = Solve[n.(OQ - p[u]) == 0, {u}] // Flatten
```

$$\left\{u \to \frac{4744}{1473}\right\}$$

```
p[u] /. solv4a
```

$$\left\{\frac{10472}{1473}, \frac{1996}{1473}, \frac{4744}{1473}\right\}$$

```
N[%]
```

```
{7.1093, 1.35506, 3.22064}
```

5

```
Remove["Global`*"]
```

a

```
Gl1 = (x + y + z == 1);
Gl2 = (2 x + 3 y + 4 z == 1);
Gl3 = (x + 2 y + 3 z == 1);
Solve[{Gl1, Gl2, Gl3}, {x, y, z}]
```

```
{}
```

Keine Lösung

b

```
Gl1 = (x + y + z == 1);
Gl2a = (2 x + 3 y + 4 z == 2);
Gl3 = (x + 2 y + 3 z == 1);
Solve[{Gl1, Gl2a, Gl3}, {x, y, z}]
```

```
{{x → 1 + z, y → -2 z}}
```

Lösung mit Dim = 1

6

```
Remove["Global`*"]
```

```
A = {{1, 2, 1}, {1, x, 2}, {-1, 2, -4}};
B = {{1, 0, 3}, {2, 3, 1}, {1, 0, 1}};
```

a

```
Det[A]
```

$$2 - 3\,x$$

```
Solve[Det[A] == 0, {x}]
```

$$\left\{\left\{x \to \frac{2}{3}\right\}\right\}$$

```
N[%]
```

$$\{\{x \to 0.666667\}\}$$

```
Det[B]
```

$$-6$$

b

```
A.B // MatrixForm
```

$$\begin{pmatrix} 6 & 6 & 6 \\ 3+2\,x & 3\,x & 5+x \\ -1 & 6 & -5 \end{pmatrix}$$

c

```
(3 Inverse[A].B /. x -> 1 / 3) // MatrixForm
```

$$\begin{pmatrix} 55 & 90 & -7 \\ -15 & -27 & 6 \\ -22 & -36 & 4 \end{pmatrix}$$

d

```
(3 B.Inverse[A] /. x -> 1 / 3) // MatrixForm
```

$$\begin{pmatrix} 5 & -6 & -4 \\ -7 & 21 & 8 \\ -9 & 18 & 6 \end{pmatrix}$$

e

```
(B.A.B.Inverse[A] /. x -> 1) // MatrixForm
```

$$\begin{pmatrix} 3 & 6 & 6 \\ 79 & -79 & -26 \\ 13 & -10 & -2 \end{pmatrix}$$

f

```
(B.Transpose[B].A.Transpose[A] /. x -> 1) // MatrixForm
```

$$\begin{pmatrix} 81 & 52 & 39 \\ 97 & 88 & -40 \\ 37 & 24 & 17 \end{pmatrix}$$

g

```
(Inverse[Transpose[Inverse[B].Inverse[A]]] /. x -> 1) // MatrixForm
```

$$\begin{pmatrix} 6 & 5 & -1 \\ 6 & 3 & 6 \\ 6 & 6 & -5 \end{pmatrix}$$

7

```
Remove["Global`*"]
```

```
M = {{2, -2}, {-3, 1}}; OP0 = {5, 8};
 M // MatrixForm
```

$$\begin{pmatrix} 2 & -2 \\ -3 & 1 \end{pmatrix}$$

```
MD[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}};
 MD[φ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

a

```
MGesamt = Inverse[M].MD[-12 Degree].M; MGesamt // MatrixForm
```

$$\begin{pmatrix} -3\left(-\frac{1}{2}\text{Cos}[12°]-\frac{1}{4}\text{Sin}[12°]\right)+2\left(-\frac{1}{4}\text{Cos}[12°]+\frac{1}{2}\text{Sin}[12°]\right) & -\frac{1}{2}\text{Cos}[12°]-2\left(-\frac{1}{4}\text{Cos} \\ -3\left(-\frac{1}{2}\text{Cos}[12°]-\frac{3}{4}\text{Sin}[12°]\right)+2\left(-\frac{3}{4}\text{Cos}[12°]+\frac{1}{2}\text{Sin}[12°]\right) & -\frac{1}{2}\text{Cos}[12°]-2\left(-\frac{3}{4}\text{Cos} \end{pmatrix}$$

```
MGesamt // N // MatrixForm
```

$$\begin{pmatrix} 1.34199 & -0.25989 \\ 0.675713 & 0.614302 \end{pmatrix}$$

b

```
OP1 = M.OP0
```

```
{-6, -7}
```

```
OP2 = MD[-12 Degree].OP1
```

$$\{-6\,\text{Cos}[12\,°] - 7\,\text{Sin}[12\,°],\ -7\,\text{Cos}[12\,°] + 6\,\text{Sin}[12\,°]\}$$

```
N[%]
```

$$\{-7.32427,\ -5.59956\}$$

```
OP3 = Inverse[M].OP2
```

$$\left\{\frac{1}{2}\,(7\,\text{Cos}[12\,°] - 6\,\text{Sin}[12\,°]) + \frac{1}{4}\,(6\,\text{Cos}[12\,°] + 7\,\text{Sin}[12\,°]),\right.$$
$$\left.-\frac{3}{4}\,(-6\,\text{Cos}[12\,°] - 7\,\text{Sin}[12\,°]) + \frac{1}{2}\,(7\,\text{Cos}[12\,°] - 6\,\text{Sin}[12\,°])\right\}$$

```
N[%]
```

$$\{4.63085,\ 8.29298\}$$

```
MGesamt.OP0 // N
```

$$\{4.63085,\ 8.29298\}$$

8

```
Remove["Global`*"]
```

```
p1 = {1, 1, 0}; p2 = {1, 0, 1}; p3 = {0, 1, 2}; s = (p1 + p2 + p3) / 3
```

$$\left\{\frac{2}{3},\ \frac{2}{3},\ 1\right\}$$

```
N[%]
```

$$\{0.666667,\ 0.666667,\ 1.\}$$

a

```
n = Cross[p2 - p1, p3 - p1]
```

$$\{-2,\ -1,\ -1\}$$

```
v = n / Norm[n]
```

$$\left\{-\sqrt{\frac{2}{3}},\ -\frac{1}{\sqrt{6}},\ -\frac{1}{\sqrt{6}}\right\}$$

```
-N[%]
```

$$\{0.816497,\ 0.408248,\ 0.408248\}$$

b

```
g[t_] := s + t n;  g[t]
```

$$\left\{\frac{2}{3} - 2\,t,\ \frac{2}{3} - t,\ 1 - t\right\}$$

```
solv8 = Solve[g[t][[3]] == 15, {t}] // Flatten
```

$$\{t \to -14\}$$

```
P4 = g[t] /. solv8
```

$$\left\{\frac{86}{3},\ \frac{44}{3},\ 15\right\}$$

```
N[%]
```

$$\{28.6667,\ 14.6667,\ 15.\}$$

c

```
M = {v, p2 - p1, p3 - p1} // Transpose;
M // MatrixForm
```

$$\begin{pmatrix} -\sqrt{\frac{2}{3}} & 0 & -1 \\ -\frac{1}{\sqrt{6}} & -1 & 0 \\ -\frac{1}{\sqrt{6}} & 1 & 2 \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.816497 & 0. & -1. \\ -0.408248 & -1. & 0. \\ -0.408248 & 1. & 2. \end{pmatrix}$$

d

```
M.(p1 + 2 (p2 - p1) + 2 (p3 - p1))
```

$$\left\{-6 + \sqrt{\frac{2}{3}},\ 1 + \frac{1}{\sqrt{6}},\ 11 + \frac{1}{\sqrt{6}}\right\}$$

```
N[%]
```

$$\{-5.1835,\ 1.40825,\ 11.4082\}$$

e

```
Norm[Cross[p2 - p1, p3 - p1]] / 2
```

$$\sqrt{\frac{3}{2}}$$

```
N[%]
```

```
1.22474
```

f

```
Norm[Cross[2 p2 - p1, 3 p3 - p1]] / 2
```

$$\frac{\sqrt{165}}{2}$$

```
N[%]
```

```
6.42262
```