Lösungen

1

$A\,A \mid \quad A^{-1}\,A^{-1}\,M\,X\,A\,A \;=\; A^{-1}\,A\,M^{-1}\,A\,A^{-1}\,A \;+\; E \quad \mid A^{-1}$

$A\,A\,A^{-1}\,A^{-1}\,M\,X\,A\,A\,A^{-1} \;=\; A\,A\;\; E\,M^{-1}\,E\,A\,A^{-1} \;+\; A\,A\;\; E\;A^{-1}$

$M^{-1} \mid \quad M\,X\,A \;=\; A\,A\,M^{-1} \;+\; A \quad \mid A^{-1}$

$M^{-1}\,M\;X \;=\; M^{-1}\,A\,A\,M^{-1}\;A^{-1} \;+\; M^{-1}\,E\,A\,A^{-1}$

$X \;=\; M^{-1}\,A\,A\,M^{-1}\;A^{-1} \;+\; M^{-1}$ oder

$X \;=\; M^{-1}\,A^2\,M^{-1}\,A^{-1} \;+\; M^{-1} \;=\; M^{-1}\,(A^2\,M^{-1}\,A^{-1} \;+\; E)$

2

```
rem; ove; Remove["Global`*"]

OA1 = {-1, 1, 1}; A1B1 = {2, 1, 1}; A1C1 = {4, 5, -2};
OA2 = {1, -1, 2}; A2B2 = {1, 2, -1}; A2C2 = {-3, 2, -4};
OQ1 = {10, -10, 12}; OQ2 = {-5, -6, 8};
v1[λ_, μ_] := OA1 + λ A1B1 + μ A1C1;
v2[ν_, σ_] := OA2 + ν A2B2 + σ A2C2;
```

a

```
VTetraeder = Det[ {OA2 + A2B2 - OA1, A1C1, OQ2 - OA1}] / 6
```

$\frac{21}{2}$

```
N[%]
```

10.5

b

```
distQ2Φ2 = Det[{ OQ2 - OA2, A2B2, A2C2}] / Norm[Cross[A2B2, A2C2]]
```

$\frac{49}{\sqrt{149}}$

```
N[%]
```

```
4.01424
```

c

```
solv01 = Solve[v1[λ, μ] == v2[ν, σ], {μ, ν, σ}] // Flatten
```

$$\left\{\mu \to \frac{3\,(6+\lambda)}{5},\ \nu \to \frac{1}{5}\,(53+13\,\lambda),\ \sigma \to -\frac{3}{5}\,(1+\lambda)\right\}$$

```
s1[λ_] := v1[λ, μ] /. solv01; s1[λ] // Simplify
```

$$\left\{\frac{1}{5}\,(67+22\,\lambda),\ 19+4\,\lambda,\ \frac{1}{5}\,(-31-\lambda)\right\}$$

```
s2[λ_] := v2[ν, σ] /. solv01; s2[λ] // Simplify
```

$$\left\{\frac{1}{5}\,(67+22\,\lambda),\ 19+4\,\lambda,\ \frac{1}{5}\,(-31-\lambda)\right\}$$

```
solv02 = Solve[s1[λ][[1]] == 0, {λ}] // Flatten
```

$$\left\{\lambda \to -\frac{67}{22}\right\}$$

```
OD1 = s1[λ] /. solv02
```

$$\left\{0,\ \frac{75}{11},\ -\frac{123}{22}\right\}$$

```
N[%]
```

```
{0., 6.81818, -5.59091}
```

```
solv03 = Solve[s1[λ][[2]] == 0, {λ}] // Flatten
```

$$\left\{\lambda \to -\frac{19}{4}\right\}$$

```
OD2 = s1[λ] /. solv03
```

$$\left\{-\frac{15}{2},\ 0,\ -\frac{21}{4}\right\}$$

```
N[%]
```

```
{-7.5, 0., -5.25}
```

```
solv04 = Solve[s1[λ][[3]] == 0, {λ}] // Flatten
```

```
{λ → -31}
```

```
OD3 = s1[λ] /. solv04
```

```
{-123, -105, 0}
```

```
N[%]
```

```
{-123., -105., 0.}
```

d

```
gQ1Q2[t_] := OQ1 + t (OQ2 - OQ1); gQ1Q2[t]
```

$$\{10 - 15\,t,\ -10 + 4\,t,\ 12 - 4\,t\}$$

```
solv05 = Solve[v1[λ, μ] == v2[ν, σ], {μ, ν, σ}] // Flatten
```

$$\left\{\mu \to \frac{3\,(6+\lambda)}{5},\ \nu \to \frac{1}{5}\,(53+13\,\lambda),\ \sigma \to -\frac{3}{5}\,(1+\lambda)\right\}$$

```
solv05 = Solve[v1[λ, μ] == gQ1Q2[t], {λ, μ, t}] // Flatten
```

$$\left\{\lambda \to \frac{363}{113},\ \mu \to -\frac{242}{113},\ t \to \frac{99}{113}\right\}$$

```
N[%]
```

$$\{\lambda \to 3.21239,\ \mu \to -2.14159,\ t \to 0.876106\}$$

```
OQ3 = gQ1Q2[t] /. solv05
```

$$\left\{-\frac{355}{113},\ -\frac{734}{113},\ \frac{960}{113}\right\}$$

```
N[%]
```

$$\{-3.14159,\ -6.49558,\ 8.49558\}$$

```
solv06 = Solve[v2[ν, σ] == gQ1Q2[t], {t, ν, σ}] // Flatten
```

$$\left\{t \to \frac{37}{86},\ \nu \to -\frac{90}{43},\ \sigma \to -\frac{133}{86}\right\}$$

```
N[%]
```

$$\{t \to 0.430233,\ \nu \to -2.09302,\ \sigma \to -1.54651\}$$

```
OQ4 = gQ1Q2[t] /. solv06
```

$$\left\{\frac{305}{86},\ -\frac{356}{43},\ \frac{442}{43}\right\}$$

```
N[%]
```

$$\{3.54651,\ -8.27907,\ 10.2791\}$$

```
Norm[OQ4 - OQ3]
```

$$\frac{4333\sqrt{257}}{9718}$$

```
N[%]
```

$$7.1479$$

e

```
winkel = ArcCos[OQ1.OQ2 / (Norm[OQ1] Norm[OQ2])]
```

$$\text{ArcCos}\left[\frac{53}{5\sqrt{430}}\right]$$

```
N[%]
```

```
1.03424
```

```
% / Degree
```

```
59.2577
```

3

```
Remove["Global`*"]
```

```
M = {{2, -2, 3}, {-3, 1, 3}, {-2, -1, 2}};
M // MatrixForm
```

$$\begin{pmatrix} 2 & -2 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 2 \end{pmatrix}$$

a

```
Det[M]
```

```
25
```

```
Det[M + M]
```

```
200
```

```
2^3 Det[M]
```

```
200
```

b

Det[M] ist nicht 0, also existiert die Inverse

c

```
invM = Inverse[M]
```

$$\left\{\left\{\frac{1}{5}, \frac{1}{25}, -\frac{9}{25}\right\}, \left\{0, \frac{2}{5}, -\frac{3}{5}\right\}, \left\{\frac{1}{5}, \frac{6}{25}, -\frac{4}{25}\right\}\right\}$$

```
invM // MatrixForm
```

$$\begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}$$

```
invM // N // MatrixForm
```

$$\begin{pmatrix} 0.2 & 0.04 & -0.36 \\ 0. & 0.4 & -0.6 \\ 0.2 & 0.24 & -0.16 \end{pmatrix}$$

d

```
transpInvM = Inverse[Transpose[M]]
```

$$\left\{\left\{\frac{1}{5}, 0, \frac{1}{5}\right\}, \left\{\frac{1}{25}, \frac{2}{5}, \frac{6}{25}\right\}, \left\{-\frac{9}{25}, -\frac{3}{5}, -\frac{4}{25}\right\}\right\}$$

```
Transpose[Inverse[M]]
```

$$\left\{\left\{\frac{1}{5}, 0, \frac{1}{5}\right\}, \left\{\frac{1}{25}, \frac{2}{5}, \frac{6}{25}\right\}, \left\{-\frac{9}{25}, -\frac{3}{5}, -\frac{4}{25}\right\}\right\}$$

```
transpInvM // MatrixForm
```

$$\begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}$$

```
transpInvM // N // MatrixForm
```

$$\begin{pmatrix} 0.2 & 0. & 0.2 \\ 0.04 & 0.4 & 0.24 \\ -0.36 & -0.6 & -0.16 \end{pmatrix}$$

e

```
{invM // MatrixForm, transpInvM // MatrixForm}
```

$$\left\{\begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}\right\}$$

```
Transpose[invM] == transpInvM
```

```
True
```

f

```
Det[invM.invM.invM.invM.invM.invM.invM]
```

$$\frac{1}{6103515625}$$

```
N[%]
```

1.6384×10^{-10}

```
Det[invM.invM.invM.invM.invM.invM.invM] == Det[invM]^7
```

True

g

```
OP0 = {2, 5, 8};  OP1 = M.OP0
```

{18, 23, 7}

```
OP2 = invM.OP0
```

$\left\{-\frac{57}{25}, -\frac{14}{5}, \frac{8}{25}\right\}$

```
N[%]
```

{−2.28, −2.8, 0.32}

h

```
M2 = M.M; M2 // MatrixForm
```

$$\begin{pmatrix} 4 & -9 & 6 \\ -15 & 4 & 0 \\ -5 & 1 & -5 \end{pmatrix}$$

```
M2.OP2 == OP1
```

True

M2 = M.M

4

```
Remove["Global`*"]
```

a

```
ga1 = (3 x + 3 y + 4 z == 2);
ga2 = (x + 2 y − z == 1);
ga3 = (4 x + 2 y + 10 z == 2);
Solve[{ga1, ga2, ga3}, {x, y, z}] // Flatten
```

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…

$\left\{x \to \frac{1}{3} - \frac{11\,z}{3}, y \to \frac{1}{3} + \frac{7\,z}{3}\right\}$

```
Solve[{ga1, ga2, ga3}, {x, y}] // Flatten
```

$$\left\{x \to \frac{1}{3}\,(1 - 11\,z),\ y \to \frac{1}{3}\,(1 + 7\,z)\right\}$$

```
N[%] // ExpandAll
```

$$\{x \to 0.333333 - 3.66667\,z,\ y \to 0.333333 + 2.33333\,z\}$$

```
Solve[{ga1, ga2, ga3}, {y, z}] // Flatten
```

$$\left\{y \to \frac{1}{11}\,(6 - 7\,x),\ z \to \frac{1}{11}\,(1 - 3\,x)\right\}$$

```
N[%] // ExpandAll
```

$$\{y \to 0.545455 - 0.636364\,x,\ z \to 0.0909091 - 0.272727\,x\}$$

```
Solve[{ga1, ga2, ga3}, {x, z}] // Flatten
```

$$\left\{x \to \frac{1}{7}\,(6 - 11\,y),\ z \to \frac{1}{7}\,(-1 + 3\,y)\right\}$$

```
N[%] // ExpandAll
```

$$\{x \to 0.857143 - 1.57143\,y,\ z \to -0.142857 + 0.428571\,y\}$$

b

```
gb1 = ga1;
gb2 = ga2;
gb3 = (4 x + 2 y + 10 z == 5);
Solve[{gb1, gb2, gb3}, {x, y, z}] // Flatten
```

$$\{\}$$

5

```
Remove["Global`*"]
```

```
OO = {0, 0, 0};
OA1 = OO;  OA2 = {4, -2, -1};  OA4 = {1, 5, 2}; OB1 = {2, 1, 10}; OC1 = OB1;
```

a

```
OA3 = OA2 + (OA4 - OA1)
```

$$\{5, 3, 1\}$$

```
OB2 = OB1 + (OA2 - OA1)
```

$$\{6, -1, 9\}$$

```
OB3 = OB2 + (OA3 - OA2)
```

$$\{7, 4, 11\}$$

```
OB4 = OB1 + (OA4 - OA1)
```

```
{3, 6, 12}
```

```
OC2 = OA2 + 0.7 (OB2 - OA2)
```

```
{5.4, -1.3, 6.}
```

```
OC4 = OA4 + 0.8 (OB4 - OA4)
```

```
{2.6, 5.8, 10.}
```

```
OC3 = OC2 + (OC4 - OC1)
```

```
{6., 3.5, 6.}
```

b

```
vSpat = Det[{OA2 - OA1, OA3 - OA1, OB1 - OA1}]
```

```
213
```

c

```
vKaese1 = Abs[Det[{OB1 - OB4, OB3 - OB4, OC4 - OB4}]] / 2
```

```
21.3
```

```
vKaese2 = Abs[Det[{OB1 - OB2, OC2 - OB2, OC3 - OC2}]] / 2
```

```
31.95
```

```
vAbgeschnitten = vKaese1 + vKaese2
```

```
53.25
```

d

```
vAbgescProzent = vAbgeschnitten / vSpat * 100
```

```
25.
```

6

```
Remove["Global`*"]
```

```
OA[1] = {-1, 0, 1};  OA[2] = {4, -2, -1};  OA[3] = {1, 5, 2};
OA[4] = {2, 1, 10}; M = {{2, -2, 3}, {-3, 1, 3}, {-2, -1, 2}};  M // MatrixForm
```

$$\begin{pmatrix} 2 & -2 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 2 \end{pmatrix}$$

a

```
vSpat = Det[{OA[2] - OA[1], OA[3] - OA[1], OA[4] - OA[3]}]
```

276

```
grundFlaeche = Norm[Cross[OA[2] - OA[1], OA[4] - OA[3]]]
```

$6\sqrt{74}$

```
abstand = vSpat / grundFlaeche
```

$$23\sqrt{\frac{2}{37}}$$

```
N[%]
```

5.34739

```
hVecSenkr = Cross[OA[2] - OA[1], OA[4] - OA[3]] / grundFlaeche abstand
```

$$\left\{-\frac{92}{37}, -\frac{161}{37}, -\frac{69}{37}\right\}$$

```
g1[t_] := OA[1] + t (OA[2] - OA[1]);
g2[s_] := OA[3] + s (OA[4] - OA[3]);
solv6 = Solve[g1[t] + c hVecSenkr == g2[s], {t, s, c}] // Flatten
```

$$\left\{t \to -\frac{8}{111}, s \to \frac{14}{111}, c \to -1\right\}$$

```
OP1 = g1[t] /. solv6
```

$$\left\{-\frac{151}{111}, \frac{16}{111}, \frac{127}{111}\right\}$$

```
N[%]
```

{-1.36036, 0.144144, 1.14414}

```
OP2 = g2[s] /. solv6
```

$$\left\{\frac{125}{111}, \frac{499}{111}, \frac{334}{111}\right\}$$

```
N[%]
```

{1.12613, 4.4955, 3.00901}

```
(g1[t] + c hVecSenkr == g2[s]) /. solv6
```

True

b

```
vTetraeder = Det[{OA[2] - OA[1], OA[3] - OA[1], OA[4] - OA[1]}] / 6
```

46

C

```
B[k_] := M.OA[k];
ta = Table[B[k], {k, 1, 4}]
```

```
{{1, 6, 4}, {9, -17, -8}, {-2, 8, -3}, {32, 25, 15}}
```

```
ta // Transpose // MatrixForm
```

$$\begin{pmatrix} 1 & 9 & -2 & 32 \\ 6 & -17 & 8 & 25 \\ 4 & -8 & -3 & 15 \end{pmatrix}$$

```
Det[{B[2] - B[1], B[3] - B[1], B[4] - B[1]}]
```

```
6900
```

```
Det[{B[2] - B[1], B[3] - B[1], B[4] - B[1]}] / 6
```

```
1150
```