Lösungen

1

```
Remove["Global`*"]
```

```
A = {{2, -2}, {-3, 1}}; A // MatrixForm
```

$$\begin{pmatrix} 2 & -2 \\ -3 & 1 \end{pmatrix}$$

```
B = {{1, 3}, {-1, 2}}; B // MatrixForm
```

$$\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$$

```
v = {{4, -3}} // Transpose
```

```
{{4}, {-3}}
```

```
A.v
```

```
{{14}, {-15}}
```

```
v = {4, -3}
```

```
{4, -3}
```

```
A.v
```

```
{14, -15}
```

- **a**

```
2 B - A // MatrixForm
```

$$\begin{pmatrix} 0 & 8 \\ 1 & 3 \end{pmatrix}$$

```
x1 = (2 B - A).v
```

```
{-24, -5}
```

- **b**

```
2 B + A // MatrixForm
```

$$\begin{pmatrix} 4 & 4 \\ -5 & 5 \end{pmatrix}$$

```
(2 B + A).{x11, x21} == v
```

```
{4 x11 + 4 x21, -5 x11 + 5 x21} == {4, -3}
```

```
((2 B + A).{x11, x21} == v) // Solve
```

$$\left\{\left\{x11 \to \frac{4}{5},\ x21 \to \frac{1}{5}\right\}\right\}$$

```
% // N
```

```
{{x11 → 0.8, x21 → 0.2}}
```

Eindeutig

2

```
Remove["Global`*"]
```

```
OA1 = {-1, 1, 1}; a = {2, 1, 1}; b = {4, 5, -2};
OA2 = {1, -1, 2}; c = {1, 2, -1}; d = {-3, 2, -4};
v1[λ_, μ_] := {-1, 1, 1} + λ {2, 1, 1} + μ {4, 5, -2};
v2[υ_, σ_] := {1, -1, 2} + υ {1, 2, -1} + σ {-3, 2, -4};
Q = {-5, -6, 8}; oO = {0, 0, 0}; OQ = Q - oO;
```

a

```
n1 = Cross[a, b]
```

```
{-7, 8, 6}
```

```
n2 = Cross[c, d]
```

```
{-6, 7, 8}
```

```
α = ArcCos[n1.n2 / (Norm[n1] Norm[n2])]
```

$$\text{ArcCos}\left[\frac{146}{149}\right]$$

```
% // N
```

```
0.201008
```

```
% / (2 Pi) 360
```

```
11.5169
```

b

```
gL1[t_] := OQ + t n1;
solv1 = Solve[v1[λ, μ] == gL1[t], {t, λ, μ}] // Flatten
```

$$\left\{t \to -\frac{14}{149},\ \lambda \to \frac{355}{149},\ \mu \to -\frac{302}{149}\right\}$$

```
OL = gL1[t] /. solv1
```

$$\left\{-\frac{647}{149},\ -\frac{1006}{149},\ \frac{1108}{149}\right\}$$

```
N[%]
```

```
{-4.34228, -6.75168, 7.43624}
```

c

```
HNF2[x_, y_, z_, dd_] = n2.{x, y, z} / Norm [n2] + dd;
(HNF2[x, y, z, dd]) == 0 /. {x → OA2[[1]] , y → OA2[[2]], z → OA2[[3]]}
```

$$\frac{3}{\sqrt{149}} + \text{dd} == 0$$

```
solv2 = Solve[%, {dd}] // Flatten;  dd = dd /. solv2
```

$$-\frac{3}{\sqrt{149}}$$

```
HNF2[{x_, y_, z_}] := HNF2[x, y, z, dd];  HNF2[OQ]
```

$$\frac{49}{\sqrt{149}}$$

```
N[%]
```

```
4.01424
```

- **d**

```
solv3[ σ_] := Solve[v1[λ, μ] == v2[υ, σ], {λ, μ, υ}] // Flatten; solv3[ σ]
```

$$\left\{\lambda \to \frac{1}{3}\ (-3-5\,\sigma),\ \mu \to 3-\sigma,\ \upsilon \to 8-\frac{13\,\sigma}{3}\right\}$$

```
((v2[υ, σ]) /. solv3[ σ]) // Simplify
```

$$\left\{9-\frac{22\,\sigma}{3},\ 15-\frac{20\,\sigma}{3},\ -6+\frac{\sigma}{3}\right\}$$

```
((v1[λ, μ]) /. solv3[ σ]) // Simplify
```

$$\left\{9-\frac{22\,\sigma}{3},\ 15-\frac{20\,\sigma}{3},\ -6+\frac{\sigma}{3}\right\}$$

```
gS[ σ_] := ((v2[υ, σ]) /. solv3[ σ]) // Simplify;
gS[ sigma]
```

$$\left\{9-\frac{22\ \text{sigma}}{3},\ 15-\frac{20\ \text{sigma}}{3},\ -6+\frac{\text{sigma}}{3}\right\}$$

```
aS = gS[ 1] - gS[0]
```

$$\left\{-\frac{22}{3},\ -\frac{20}{3},\ \frac{1}{3}\right\}$$

```
Flaeche = Norm[Cross[gS[ 1] - gS[0], OQ - gS[0] ]]
```

$$\frac{7\sqrt{3809}}{3}$$

```
h = Flaeche / Norm[aS]
```

$$7\sqrt{\frac{3809}{885}}$$

```
N[%]
```

```
14.5222
```

3

```
Remove["Global`*"]
```

```
a1T[r_] := {r, -3, -2};  a2T = {-1, 1, -1}; a3T = {3, 3, 0};
```

```
A[r_] := Transpose[{a1T[r], a2T, a3T}]; A[r] // MatrixForm
```

$$\begin{pmatrix} r & -1 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 0 \end{pmatrix}$$

- **a**

```
Det[A[r]]
```

```
21 + 3 r
```

- **b**

```
Solve[Det[A[r]] == 0]
```

```
{{r → -7}}
```

- **c**

  r ungleich - 7

- **d**

```
Solve[Det[A[r]] == 1]
```

$$\left\{\left\{r \to -\frac{20}{3}\right\}\right\}$$

```
% // N
```

```
{{r → -6.66667}}
```

4

```
Remove["Global`*"]
```

```
a1T[r_] := {r, -3, -2};  a2T = {-1, 1, -1}; a3T = {3, 3, 0};
```

```
A[r_] := Transpose[{a1T[r], a2T, a3T}]; A[r] // MatrixForm
```

$$\begin{pmatrix} r & -1 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 0 \end{pmatrix}$$

- **a**

```
Vwuerfel = s^3
```

$$s^3$$

```
Voktaeder = 8 × 1 / 6 (s / 2) ^3
```

$$\frac{s^3}{6}$$

```
Vwuerfel / Voktaeder
```

```
6
```

- **b**

```
E1 = {0, 1, 1}; E2 = {1, 0, 1}; E3 = {1, 1, 0};
E4 = {2, 1, 1}; E5 = {1, 2, 1}; E6 = {1, 1, 2};
```

```
V = Det[{E2 - E1, E3 - E1, E4 - E1}]
```

```
2
```

```
V = Det[{E2 - E1, E3 - E1, E5 - E1}]
```

```
2
```

```
Fl = Norm[Cross[E2 - E1, E3 - E1]]
```

$$\sqrt{3}$$

```
V / Fl
```

$$\frac{2}{\sqrt{3}}$$

```
N[%]
```

```
1.1547
```

c

```
n1 = Cross[E3 - E1, E2 - E1]
```

```
{-1, -1, -1}
```

```
n2 = Cross[E6 - E1, E2 - E1]
```

```
{1, 1, -1}
```

```
α = ArcCos[n1.n2 / (Norm[n1] Norm[n2])]
```

$$\text{ArcCos}\left[-\frac{1}{3}\right]$$

```
% // N
```

```
1.91063
```

```
Pi - %
```

```
1.23096
```

```
% / (2 Pi) 360
```

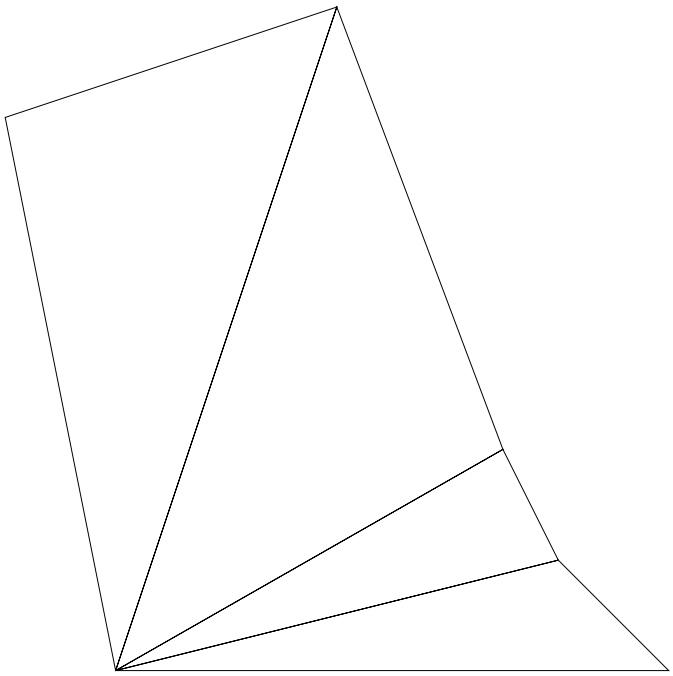
```
70.5288
```

5

```
Remove["Global`*"]
```

Darstellung nur Dreiecke

```
P0 = {0, 0}; P1 = {5, 0}; P2 = {4, 1}; P3 = {3.5, 2}; P4 = {2, 6}; P5 = {-1, 5};
Show[Graphics[Line[{P0, P1, P2, P0, P2, P3, P0, P3, P4, P0, P4, P5, P0}]]]
```



Nur Dreiecke statt Parallelogramme gezeigt.

a

```
Fl[A_, B_] := A[[1]] B[[2]] - A[[2]] B[[1]];
```

```
Fl[P1, P2]
```

```
5
```

```
Fl[P2, P3]
```

```
4.5
```

```
Fl[P3, P4]
```

```
17.
```

```
Fl[P4, P5]
```

```
16
```

```
Fl[P1, P2] + Fl[P2, P3] + Fl[P3, P4] + Fl[P4, P5]
```

```
42.5
```

- **b**

```
MD[ϕ_] := {{Cos[ϕ], -Sin[ϕ]}, {Sin[ϕ], Cos[ϕ]}};  MatrixForm[MD[ϕ]]
```

$$\begin{pmatrix} \text{Cos}[\phi] & -\text{Sin}[\phi] \\ \text{Sin}[\phi] & \text{Cos}[\phi] \end{pmatrix}$$

```
MatrixForm[MD[34.7 Degree]]
```

$$\begin{pmatrix} 0.822144 & -0.56928 \\ 0.56928 & 0.822144 \end{pmatrix}$$

- **c**

```
P6 = MD[34.7 Degree].P5
```

```
{-3.66854, 3.54144}
```

6

```
Remove["Global`*"]
```

```
z1 = 1 - 2 I;  z2 = -2 - 3 I;  z3 = -1 - 5 I;
```

- **a**

```
w1 = 5 z1^2 - z2 / z3
```

$$-\frac{407}{26} - \frac{513\,i}{26}$$

```
w1 // N
```

```
-15.6538 - 19.7308 i
```

- **b**

```
Solve[z^5 == z1, {z}]
```

$$\left\{\left\{z \to (1-2\,i)^{1/5}\right\}, \left\{z \to -(-1)^{1/5}\,(1-2\,i)^{1/5}\right\}, \left\{z \to (-1)^{2/5}\,(1-2\,i)^{1/5}\right\}, \left\{z \to -(-1)^{3/5}\,(1-2\,i)^{1/5}\right\}, \left\{z \to (-1)^{4/5}\,(1-2\,i)^{1/5}\right\}\right\}$$

```
M = Solve[z^5 == z1, {z}] // N // Flatten
```

```
{z → 1.14594 - 0.257975 i, z → -1.07872 - 0.46486 i,
 z → 0.599464 + 1.01013 i, z → 0.108766 - 1.16957 i, z → -0.775451 + 0.882273 i}
```

■ **c**

```
zz[k_] := z /. M[[k]];
zz[1]
```

```
1.14594 - 0.257975 i
```

```
Sum[zz[k], {k, 1, 5}] // Chop
```

```
0
```

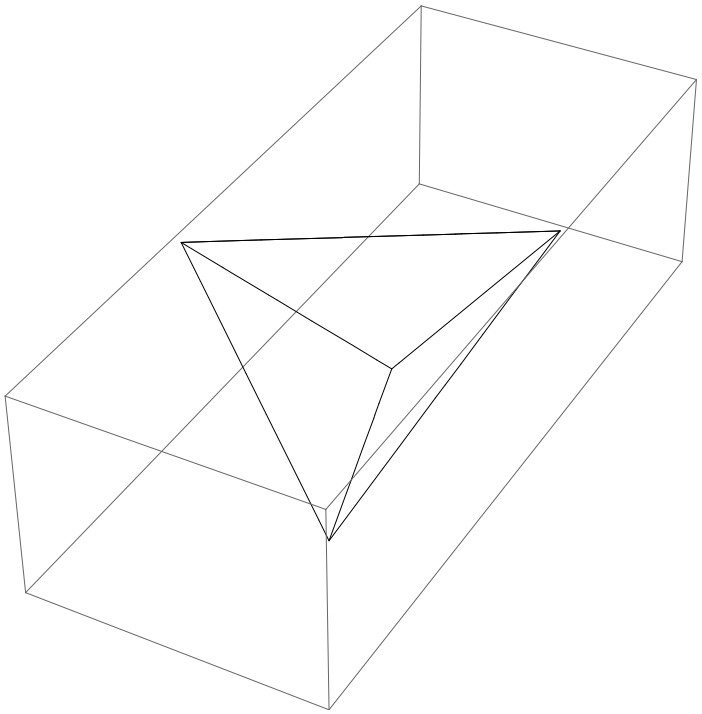
7

```
Remove["Global`*"]
```

```
PA = {2, 1, 0}; PB = {3, 8, 0}; PC = {-1, -3, 5}; PD = {6, -9, 4};
```

■

```
Show[Graphics3D[Line[{PA, PB, PC, PA, PD, PB, PC, PD}]]]
```



```
Det[{PB - PA, PC - PA, PD - PA}]
```

```
258
```

```
Det[{PB - PA, PC - PA, PD - PA}] == 0
```

```
False
```

==> Tetraeder

■

```
Fl[a_, b_] := Norm[1 / 2 Cross[a, b]] // N
```

```
Fl[PB - PA, PC - PA]
```

```
19.615
```

```
Fl[PD - PA, PC - PA]
```

```
32.7719
```

```
Fl[PD - PA, PB - PA]
```

```
23.6854
```

```
Fl[PB - PD, PC - PD]
```

```
56.6635
```

```
Fl[PB - PA, PC - PA] + Fl[PD - PA, PC - PA] + Fl[PD - PA, PB - PA] + Fl[PB - PD, PC - PD]
```

```
132.736
```

8

```
Remove["Global`*"]
```

```
M = {2, 1, 5};  r = 4;  T[z_] := {1, -0.5, z}
```

■

```
(M - T[z]).(M - T[z]) == r^2
```

$$3.25 + (5 - z)^2 == 16$$

```
solv = Solve[(M - T[z]).(M - T[z]) == r^2, {z}] // Flatten
```

$$\{z \to 1.42929,\ z \to 8.57071\}$$

```
T[z] = T[z] /. solv[[1]]
```

$$\{1, -0.5, 1.42929\}$$

```
((M - {x, y, z}).(M - T[z]) == r^2) // Simplify
```

$$1.\ x + 1.5\ y + 3.57071\ z == 5.35357$$

```
(((M - {x, y, z}).(M - T[z]) == r^2) /. {y → 0, z → 0}) // Simplify
```

$$1.\ x == 5.35357$$