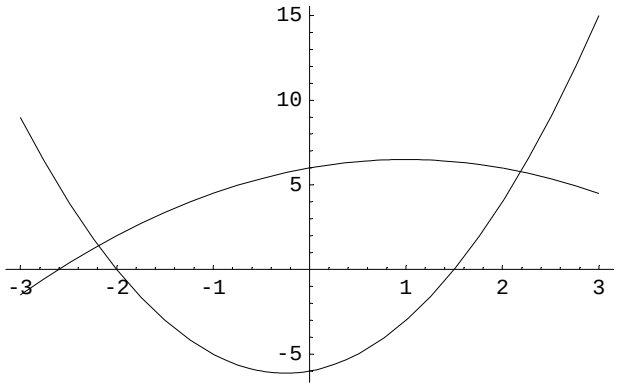
Lösungen Test 3 1. Semester B1

1 Skizze?

```
f[x_]:=-1/2 x^2+x+6;
g[x_]:=2 x^2+x-6;
Plot[{f[x],g[x]},{x,-3,3}];
```




```
s1=Solve[f[x]==g[x],{x}]//Flatten
```

$$\left\{x \to -2\sqrt{\frac{6}{5}},\ x \to 2\sqrt{\frac{6}{5}}\right\}$$

```
N[s1]
```

```
{x → -2.19089, x → 2.19089}
```

```
x1=x/.s1[[1]];x2=x/.s1[[2]];
```

```
Integrate[f[x]-g[x],{x,x1,x2}]
```

$$32\sqrt{\frac{6}{5}}$$

```
N[%]
```

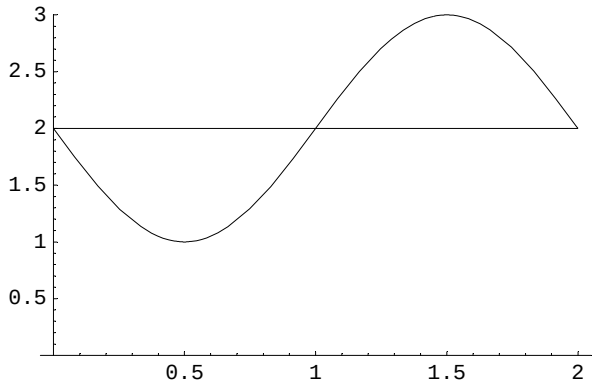
```
35.0542
```

2 Skizze?

```
Integrate[2-Sin[Pi x],{x,0,2}]
```

```
4
```

```
Plot[{2, 2-Sin[Pi x]},{x,0,2},PlotRange->{0,3}];
```



3 Bestimme die Stammfunktion

```
f[x_]:=Sinh[30x-15]; f[x]
```

$$-\text{Sinh}[15-30\,x]$$

```
Integrate[f[x],{x,-20,20}]
```

$$-\frac{1}{15}\,\text{Sinh}[15]\,\text{Sinh}[600]$$

```
Integrate[f[x],{x,-20,20}]//N
```

$$-2.05568\times 10^{265}$$

4 Lässt sich die Stammfunktion bestimmen? Lässt sich das Integral rechnen?

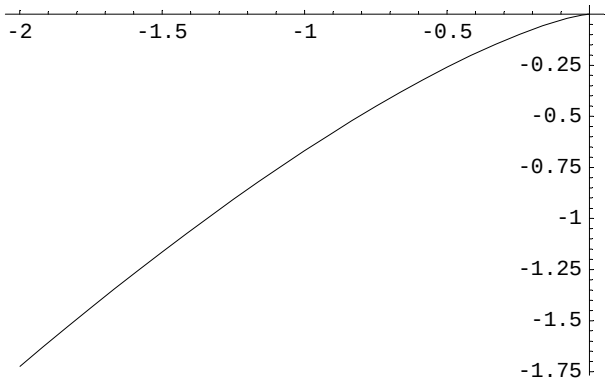
In[1]:=
```
f[x_]:=x^(2/3) (-x)^(1/2)/x^(4/5); f[x]
```

Out[1]=
$$\frac{\sqrt{-x}}{x^{2/15}}$$

In[3]:=
```
Integrate[f[x],x]//Simplify
```

Out[3]=
$$\frac{30}{41}\sqrt{-x}\;x^{13/15}$$

In[5]:= **Plot[Evaluate[Re[Integrate[f[x],x]]],{x,0,-2}];**

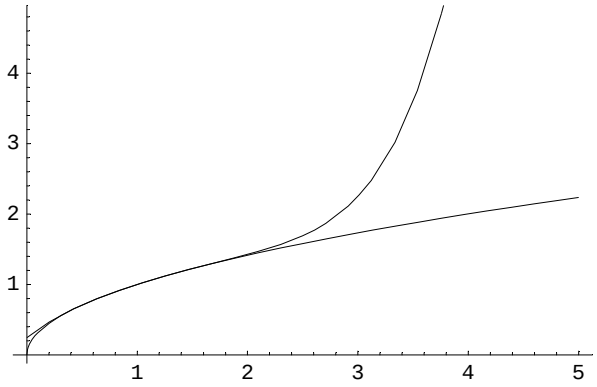


5 Skizze, berechne die Abweichung der beiden Flächen

f[x_] := Normal $\left[\text{Series}\left[\sqrt{x}, \{x, 1, 5\}\right]\right]$; f[x]

$$1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4 + \frac{7}{256}(-1+x)^5$$

Plot[Evaluate[{f[x], Sqrt[x]}], {x, 0, 5}];



{Integrate[f[x], {x, 0, 1}], Integrate[Sqrt[x], {x, 0, 1}]}

$$\left\{\frac{1045}{1536}, \frac{2}{3}\right\}$$

%//N

{0.680339, 0.666667}

Integrate$\left[\text{Evaluate}\left[\text{Normal}\left[\text{Series}\left[\sqrt{x}, \{x, 1, 5\}\right]\right] - \text{Sqrt}[x]\right], \{x, 0, 1\}\right]$

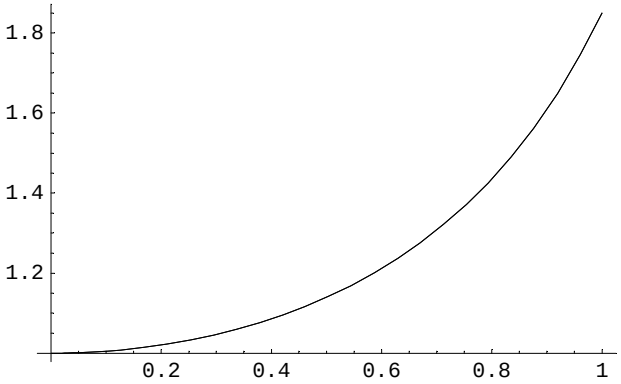
$$\frac{7}{512}$$

%//N

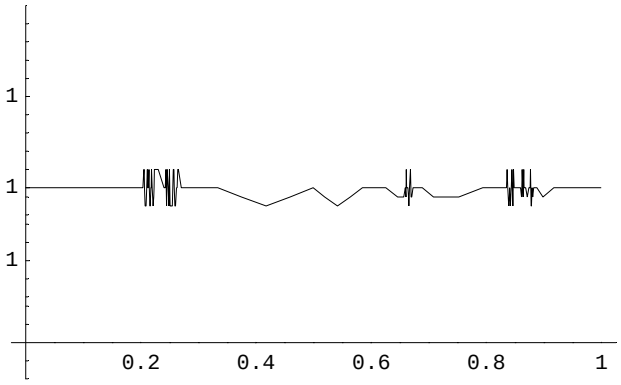
0.0136719

6 Skizze, berechne

```
Plot[{Sec[x],1/Cos[x]},{x,0,1}];
```




```
Plot[{Sec[x]^2/(1+Tan[x]^2)},{x,0,1}];
```




```
(Cos[x]^2 (1+b^2 Sin[x]^2/Cos[x]^2))//ExpandAll
```

$$\text{Cos}[x]^2 + b^2\,\text{Sin}[x]^2$$

```
D[ArcTan[b Tan[x]],x]
```

$$\frac{b\,\text{Sec}[x]^2}{1 + b^2\,\text{Tan}[x]^2}$$

```
Integrate[-1/(1+Cos[x+α]^2),x]//Simplify
```

$$-\frac{\text{ArcTan}\left[\frac{\text{Tan}[x+\alpha]}{\sqrt{2}}\right]}{\sqrt{2}}$$

```
(Integrate[-1/(1+Cos[x+α]^2),x]/.x->-α)-(Integrate[-1/(1+Cos[x+α]^2),x]/.x->0)//Simplify
```

$$\frac{\text{ArcTan}\left[\frac{\text{Tan}[\alpha]}{\sqrt{2}}\right]}{\sqrt{2}}$$

7 Skizze, berechne die Partialbruchzerlegung und eine Stammfunktion

Apart[(-3x^3+3x^2-x+1)/(x^2-x)]

$$-\frac{1}{x}-3x$$

Integrate[(-3x^3+3x^2-x+1)/(x^2-x),x]

$$-\frac{3x^2}{2}-\text{Log}[x]$$

8 Integriere von Hand

Integrate[x^2 Log[x],x]

$$-\frac{x^3}{9}+\frac{1}{3}x^3\,\text{Log}[x]$$

9 Stammfunktion, integrieren

Integrate[Cos[x] Sin[Sin[x]],x]

$$-\text{Cos}[\text{Sin}[x]]$$

Integrate[Cos[x] Sin[Sin[x]],{x,0,Pi}]

0

10 Stammfunktion, integrieren, exakt

Integrate[Cos[x]/E^Sin[x],x]

$$-e^{-\text{Sin}[x]}$$

Integrate[Cos[x]/E^Sin[x],{x,0,Pi/2}]

$$\frac{-1+e}{e}$$

Integrate[Cos[x]/E^Sin[x],{x,0,Pi/2}]//N

0.632121

NIntegrate[Cos[x]/E^Sin[x],{x,0,Pi/2}]

0.632121

11 a Länge Linie E^x

Integrate[Sqrt[1+E^(2x)],x]

$$\sqrt{1+e^{2x}}-\text{ArcTanh}\left[\sqrt{1+e^{2x}}\right]$$

Integrate[Sqrt[1+E^(2x)],{x,0,1}]

$$-\sqrt{2}+\sqrt{1+e^{2}}+\text{ArcTanh}\left[\sqrt{2}\right]-\text{ArcTanh}\left[\sqrt{1+e^{2}}\right]$$

Integrate[Sqrt[1+E^(2x)],{x,0,1}]//N//Chop

2.0035

11 b Volumen des Rotationskörpers

Integrate[Pi E^(2x),{x,0,1}]

$$\frac{1}{2}\left(-1+e^{2}\right)\pi$$

%//N

10.0359

Limit[Integrate[Pi E^(-2x),{x,0,a}],a->Infinity]

$$\frac{\pi}{2}$$

11 c Oberfläche des Rotationskörpers

Integrate[2 Pi E^x Sqrt[1+E^(2 x)],x]

$$\pi\left(e^{x}\sqrt{1+e^{2x}}+\text{ArcSinh}[e^{x}]\right)$$

Integrate[2 Pi E^x Sqrt[1+E^(2 x)],{x,0,1}]

$$\pi\left(-\sqrt{2}+e\sqrt{1+e^{2}}-\text{ArcSinh}[1]+\text{ArcSinh}[e]\right)$$

%//N

22.943

Limit[Integrate[2 Pi E^x Sqrt[1+E^(2 x)],{x,-a,0}],a->Infinity]

$$\pi\left(\sqrt{2}+\text{ArcSinh}[1]\right)$$

%//N

7.2118