Lösungen

1

```
Remove["Global`*"]
```

```
M = {6, 5};
Pk[x_, y_] := {x, y};
r = 4;
Kr[x_, y_] := (Pk[x, y] - M).(Pk[x, y] - M) - r^2;
Kr[{x_, y_}] := Kr[x, y];
P = {-1, 0};
```

- **b**

```
d1 = Norm[M - P] - r
```

$-4+\sqrt{74}$

```
% // N
```

4.60233

```
d2 = Norm[M - P] + r
```

$4+\sqrt{74}$

```
% // N
```

12.6023

```
{Kr[x, y] == 0, (Pk[x, y] - P).(Pk[x, y] - P) == d1 d2} // Simplify
```

$\{(-6+x)^2+(-5+y)^2 == 16,\ (1+x)^2+y^2 == 58\}$

```
solv = Solve[{Kr[x, y] == 0, (Pk[x, y] - P).(Pk[x, y] - P) == d1 d2}, {x, y}]
```

$\left\{\left\{x\to\frac{2}{37}\left(83-5\sqrt{58}\right),\ y\to\frac{1}{37}\left(145+14\sqrt{58}\right)\right\},\ \left\{x\to\frac{2}{37}\left(83+5\sqrt{58}\right),\ y\to\frac{1}{37}\left(145-14\sqrt{58}\right)\right\}\right\}$

```
T1 = {x, y} /. solv[[2]]
```

$\left\{\frac{2}{37}\left(83+5\sqrt{58}\right),\ \frac{1}{37}\left(145-14\sqrt{58}\right)\right\}$

```
N[%]
```

{6.5448, 1.03728}

```
T2 = {x, y} /. solv[[1]]
```

$\left\{\frac{2}{37}\left(83-5\sqrt{58}\right),\ \frac{1}{37}\left(145+14\sqrt{58}\right)\right\}$

```
N[%]
```

{2.42817, 6.80056}

```
S1 = P + (M - P) / Norm[M - P] d1
```

$\left\{-1+\frac{7\left(-4+\sqrt{74}\right)}{\sqrt{74}},\ \frac{5\left(-4+\sqrt{74}\right)}{\sqrt{74}}\right\}$

```
N[%]
```

{2.74507, 2.67505}

```
S2 = P + (M - P) / Norm[M - P] d2
```

$$\left\{-1+\frac{7\left(4+\sqrt{74}\right)}{\sqrt{74}},\ \frac{5\left(4+\sqrt{74}\right)}{\sqrt{74}}\right\}$$

```
N[%]
```

{9.25493, 7.32495}

```
Q = T1 + (T2 - T1) / 3  2
```

$$\left\{\frac{2}{37}\left(83+5\sqrt{58}\right)+\frac{2}{3}\left(\frac{2}{37}\left(83-5\sqrt{58}\right)-\frac{2}{37}\left(83+5\sqrt{58}\right)\right),\ \frac{1}{37}\left(145-14\sqrt{58}\right)+\frac{2}{3}\left(\frac{1}{37}\left(-145+14\sqrt{58}\right)+\frac{1}{37}\left(145+14\sqrt{58}\right)\right)\right\}$$

```
N[%]
```

{3.80038, 4.87947}

- **C**

```
gPQ[t_] := P + t (Q - P);
```

```
{Kr[x, y] == 0, Pk[x, y] == gPQ[t]}
```

$$\left\{-16+(-6+x)^2+(-5+y)^2==0,\ \{x,y\}==\left\{-1+\left(1+\frac{2}{37}\left(83+5\sqrt{58}\right)+\frac{2}{3}\left(\frac{2}{37}\left(83-5\sqrt{58}\right)-\frac{2}{37}\left(83+5\sqrt{58}\right)\right)\right)t,\ \left(\frac{1}{37}\left(145-14\sqrt{58}\right)+\frac{2}{3}\left(\frac{1}{37}\left(-145+14\sqrt{58}\right)+\frac{1}{37}\left(145+14\sqrt{58}\right)\right)\right)t\right\}\right\}$$

```
solv2 = Solve[{Kr[x, y] == 0, Pk[x, y] == gPQ[t]}, {x, y, t}]
```

$$\left\{\left\{x\to\frac{2\left(28823-2436\sqrt{37}-555\sqrt{58}+40\sqrt{2146}\right)}{9953},\ y\to\frac{48285-3480\sqrt{37}+1554\sqrt{58}-112\sqrt{2146}}{9953},\ t\to\frac{3}{269}\left(111-8\sqrt{37}\right)\right\},\ \left\{x\to\frac{2\left(28823+2436\sqrt{37}-555\sqrt{58}-40\sqrt{2146}\right)}{9953},\ y\to\frac{48285+3480\sqrt{37}+1554\sqrt{58}+112\sqrt{2146}}{9953},\ t\to\frac{3}{269}\left(111+8\sqrt{37}\right)\right\}\right\}$$

```
N[%]
```

{{x → 2.33731, y → 3.39229, t → 0.695218}, {x → 7.54765, y → 8.68847, t → 1.78062}}

```
H1 = {x, y} /. solv2[[1]]
```

$$\left\{\frac{2\left(28823-2436\sqrt{37}-555\sqrt{58}+40\sqrt{2146}\right)}{9953},\ \frac{48285-3480\sqrt{37}+1554\sqrt{58}-112\sqrt{2146}}{9953}\right\}$$

```
N[%]
```

{2.33731, 3.39229}

```
H2 = {x, y} /. solv2[[2]]
```

$$\left\{\frac{2\left(28823+2436\sqrt{37}-555\sqrt{58}-40\sqrt{2146}\right)}{9953},\ \frac{48285+3480\sqrt{37}+1554\sqrt{58}+112\sqrt{2146}}{9953}\right\}$$

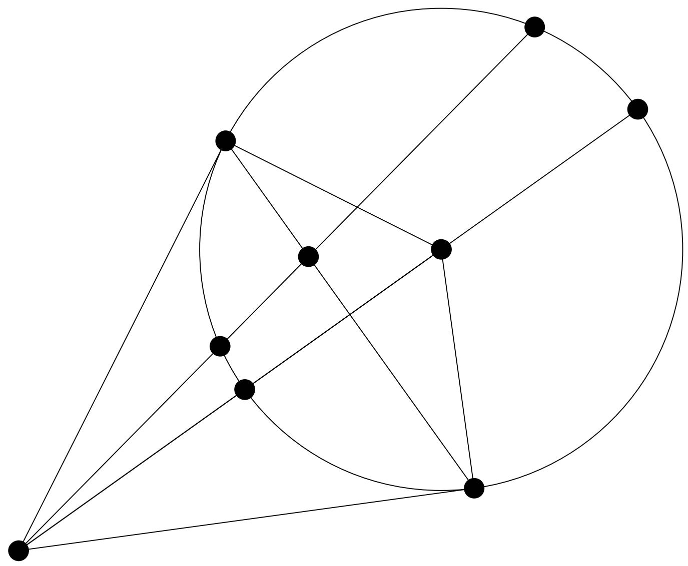
```
N[%]
```

{7.54765, 8.68847}

a

```
Show[Graphics[{
   Circle[M, r],
   PointSize[0.03], Point[M], Point[P], Point[T1], Point[T2],
   Point[P + (M - P) / Norm[M - P] d2], Point[S1],
   Point[Q], Point[H1], Point[H2],
   Line[{M, T1, P, M, T2, P}], Line[{P, S2}],
   Line[{T1, T2}], Line[{P, H2}]

  }]]
```



d

```
Norm[T1 - P]^2
```

$$\frac{\left(145-14\sqrt{58}\right)^2}{1369}+\left(1+\frac{2}{37}\left(83+5\sqrt{58}\right)\right)^2$$

```
N[%]
```

58.

```
Norm[T2 - P]^2
```

$$\frac{\left(145+14\sqrt{58}\right)^2}{1369}+\left(1+\frac{2}{37}\left(83-5\sqrt{58}\right)\right)^2$$

```
N[%]
```

58.

```
Norm[H1 - P] Norm[H2 - P]
```

$$\sqrt{\left(\left(\frac{\left(48285+3480\sqrt{37}+1554\sqrt{58}+112\sqrt{2146}\right)^2}{99062209}+\left(1+\frac{2\left(28823+2436\sqrt{37}-555\sqrt{58}-40\sqrt{2146}\right)}{9953}\right)^2\right)\left(\frac{\left(48285-3480\sqrt{37}+1554\sqrt{58}-112\sqrt{2146}\right)^2}{99062209}+\left(1+\frac{2\left(28823-2436\sqrt{37}-555\sqrt{58}+40\sqrt{2146}\right)}{9953}\right)^2\right)\right)}$$

```
N[%]
```

58.

2

```
Remove["Global`*"]
```

a

```
B = {{r, 2}, {3, 2}}; B // MatrixForm
```

$$\begin{pmatrix} r & 2 \\ 3 & 2 \end{pmatrix}$$

```
Inverse[B] // MatrixForm
```

$$\begin{pmatrix} \frac{2}{-6+2r} & -\frac{2}{-6+2r} \\ -\frac{3}{-6+2r} & \frac{r}{-6+2r} \end{pmatrix}$$

```
% // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{1}{-3+r} & \frac{1}{3-r} \\ \frac{3}{6-2r} & \frac{r}{-6+2r} \end{pmatrix}$$

b

```
A[r_] := {{r, 2, 3 r}, {3, 2, 1}, {1, 3, -2}}; A[r] // MatrixForm
```

$$\begin{pmatrix} r & 2 & 3r \\ 3 & 2 & 1 \\ 1 & 3 & -2 \end{pmatrix}$$

```
Det[A[r]]
```

$14 + 14 r$

c

```
Inverse[A[r]] // MatrixForm
```

$$\begin{pmatrix} -\frac{7}{14+14r} & \frac{4+9r}{14+14r} & \frac{2-6r}{14+14r} \\ \frac{7}{14+14r} & -\frac{5r}{14+14r} & \frac{8r}{14+14r} \\ \frac{7}{14+14r} & \frac{2-3r}{14+14r} & \frac{-6+2r}{14+14r} \end{pmatrix}$$

```
Solve[Det[A[r]] == 0, {r}]
```

```
{{r → -1}}
```

■ d

A (1) X Inv[A (1)] + E = A (0) - A (r) X Inv[A (1)]
A (1) X Inv[A (1)] + A (r) X Inv[A (1)] = A (0) - E
(A (1) + A (r)) X Inv[A (1)] = (A (0) - E) | von rechts mal A (1)
von links mal Inv[(A (1) + A (r))] | (A (1) + A (r)) X E = (A (0) - E) A (1)
E X = Inv[(A (1) + A (r))] (A (0) - E) A (1)
Resultat :
X = Inv[(A (1) + A (r))] (A (0) - E) A (1)
=== === === === === === === === === === ==

```
X[x11_, x12_, x13_, x21_, x22_, x23_, x31_, x32_, x33_] :=
  {{x11, x12, x13}, {x21, x22, x23}, {x31, x32, x33}};
X[x11, x12, x13, x21, x22, x23, x31, x32, x33] // MatrixForm
```

$$\begin{pmatrix} x11 & x12 & x13 \\ x21 & x22 & x23 \\ x31 & x32 & x33 \end{pmatrix}$$

```
A[1].X[x11, x12, x13, x21, x22, x23, x31, x32, x33].Inverse[A[1]] + IdentityMatrix[3] ==
 A[0] - A[r].X[x11, x12, x13, x21, x22, x23, x31, x32, x33].Inverse[A[1]]
```

$$\Big\{\Big\{1+\frac{1}{4}(-x11-2\,x21-3\,x31)+\frac{1}{4}(x12+2\,x22+3\,x32)+\frac{1}{4}(x13+2\,x23+3\,x33),$$
$$\frac{13}{28}(x11+2\,x21+3\,x31)-\frac{5}{28}(x12+2\,x22+3\,x32)+\frac{1}{28}(-x13-2\,x23-3\,x33),$$
$$\frac{1}{7}(-x11-2\,x21-3\,x31)+\frac{2}{7}(x12+2\,x22+3\,x32)+\frac{1}{7}(-x13-2\,x23-3\,x33)\Big\},$$
$$\Big\{\frac{1}{4}(-3\,x11-2\,x21-x31)+\frac{1}{4}(3\,x12+2\,x22+x32)+\frac{1}{4}(3\,x13+2\,x23+x33),$$
$$1+\frac{13}{28}(3\,x11+2\,x21+x31)-\frac{5}{28}(3\,x12+2\,x22+x32)+\frac{1}{28}(-3\,x13-2\,x23-x33),$$
$$\frac{1}{7}(-3\,x11-2\,x21-x31)+\frac{2}{7}(3\,x12+2\,x22+x32)+\frac{1}{7}(-3\,x13-2\,x23-x33)\Big\},$$
$$\Big\{\frac{1}{4}(-x11-3\,x21+2\,x31)+\frac{1}{4}(x12+3\,x22-2\,x32)+\frac{1}{4}(x13+3\,x23-2\,x33),$$
$$\frac{13}{28}(x11+3\,x21-2\,x31)-\frac{5}{28}(x12+3\,x22-2\,x32)+\frac{1}{28}(-x13-3\,x23+2\,x33),$$
$$1+\frac{1}{7}(-x11-3\,x21+2\,x31)+\frac{2}{7}(x12+3\,x22-2\,x32)+\frac{1}{7}(-x13-3\,x23+2\,x33)\Big\}\Big\}==$$
$$\Big\{\Big\{\frac{1}{4}(r\,x11+2\,x21+3\,r\,x31)+\frac{1}{4}(-r\,x12-2\,x22-3\,r\,x32)+\frac{1}{4}(-r\,x13-2\,x23-3\,r\,x33),$$
$$2-\frac{13}{28}(r\,x11+2\,x21+3\,r\,x31)+\frac{5}{28}(r\,x12+2\,x22+3\,r\,x32)+\frac{1}{28}(r\,x13+2\,x23+3\,r\,x33),$$
$$\frac{1}{7}(r\,x11+2\,x21+3\,r\,x31)-\frac{2}{7}(r\,x12+2\,x22+3\,r\,x32)+\frac{1}{7}(r\,x13+2\,x23+3\,r\,x33)\Big\},$$
$$\Big\{3+\frac{1}{4}(3\,x11+2\,x21+x31)+\frac{1}{4}(-3\,x12-2\,x22-x32)+\frac{1}{4}(-3\,x13-2\,x23-x33),$$
$$2-\frac{13}{28}(3\,x11+2\,x21+x31)+\frac{5}{28}(3\,x12+2\,x22+x32)+\frac{1}{28}(3\,x13+2\,x23+x33),$$
$$1+\frac{1}{7}(3\,x11+2\,x21+x31)-\frac{2}{7}(3\,x12+2\,x22+x32)+\frac{1}{7}(3\,x13+2\,x23+x33)\Big\},$$
$$\Big\{1+\frac{1}{4}(x11+3\,x21-2\,x31)+\frac{1}{4}(-x12-3\,x22+2\,x32)+\frac{1}{4}(-x13-3\,x23+2\,x33),$$
$$3-\frac{13}{28}(x11+3\,x21-2\,x31)+\frac{5}{28}(x12+3\,x22-2\,x32)+\frac{1}{28}(x13+3\,x23-2\,x33),$$
$$-2+\frac{1}{7}(x11+3\,x21-2\,x31)-\frac{2}{7}(x12+3\,x22-2\,x32)+\frac{1}{7}(x13+3\,x23-2\,x33)\Big\}\Big\}$$

```
solv3 = Solve[
   A[1].X[x11, x12, x13, x21, x22, x23, x31, x32, x33].Inverse[A[1]] + IdentityMatrix[3] ==
    A[0] - A[r].X[x11, x12, x13, x21, x22, x23, x31, x32, x33].Inverse[A[1]],
   { x11, x12, x13, x21, x22, x23, x31, x32, x33}] // Flatten
```

$$\Big\{x11\to-\frac{-5-3\,r}{4\,(3+r)},\ x12\to-\frac{-23-15\,r}{4\,(3+r)},\ x13\to\frac{9}{2\,(3+r)},\ x21\to\frac{3}{2}+\frac{-5-3\,r}{4\,(3+r)},\ x22\to-\frac{5+9\,r}{4\,(3+r)},$$
$$x23\to-\frac{-3-4\,r}{2\,(3+r)},\ x31\to\frac{1}{2}+\frac{-5-3\,r}{4\,(3+r)},\ x32\to-\frac{-7+5\,r}{4\,(3+r)},\ x33\to-\frac{9}{2\,(3+r)}\Big\}$$

```
Xres[r_] := X[x11, x12, x13, x21, x22, x23, x31, x32, x33] /. solv3;
Xres[r] // MatrixForm
```

$$\begin{pmatrix} -\frac{-5-3 r}{4 (3+r)} & -\frac{-23-15 r}{4 (3+r)} & \frac{9}{2 (3+r)} \\ \frac{3}{2}+\frac{-5-3 r}{4 (3+r)} & -\frac{5+9 r}{4 (3+r)} & -\frac{-3-4 r}{2 (3+r)} \\ \frac{1}{2}+\frac{-5-3 r}{4 (3+r)} & -\frac{-7+5 r}{4 (3+r)} & -\frac{9}{2 (3+r)} \end{pmatrix}$$

- **e**

```
(Xres[r] /. r → 2) // MatrixForm
```

$$\begin{pmatrix} \frac{11}{20} & \frac{53}{20} & \frac{9}{10} \\ \frac{19}{20} & -\frac{23}{20} & \frac{11}{10} \\ -\frac{1}{20} & -\frac{3}{20} & -\frac{9}{10} \end{pmatrix}$$

```
N[%]
```

{{0.55, 2.65, 0.9}, {0.95, -1.15, 1.1}, {-0.05, -0.15, -0.9}}

```
{{0.55`, 2.65`, 0.9`}, {0.95`, -1.15`, 1.1`}, {-0.05`, -0.15`, -0.9`}} // MatrixForm
```

$$\begin{pmatrix} 0.55 & 2.65 & 0.9 \\ 0.95 & -1.15 & 1.1 \\ -0.05 & -0.15 & -0.9 \end{pmatrix}$$

```
(Xres[r] /. r → -3) // MatrixForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

$$\begin{pmatrix} \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \\ \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \\ \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \end{pmatrix}$$

Existiert nicht!

3

- **Vorbemerkung: Es gibt 2 reelle Lösungen. Der Rechte Winkel kann bei A, B oder C liegen. Im Falle dass er bei A liegt, gibt es keine reelle Lösung.**

3.1 Rechter Winkel bei B

```
Remove["Global`*"]
```

- **a**

```
pA[y_] := {16, y, 0}; pB = {13, 7, 12}; pC = {2, 2, 8};
{pA[y], pB, pC}
```

{{16, y, 0}, {13, 7, 12}, {2, 2, 8}}

```
solve4 = Solve[(pB - pA[y]).(pC - pB) == 0, {y}] // Flatten
```

$\{y \to 10\}$

```
pA1 = pA[y] /. solve4
```

{16, 10, 0}

- **b**

```
{Norm[pB - pA1], Norm[pC - pB], Norm[pC - pA1]}
```

$\left\{9\sqrt{2}, 9\sqrt{2}, 18\right\}$

Gleichschenklig

- **c**

```
pD = pA1 + (pC - pB)
```

{5, 5, -4}

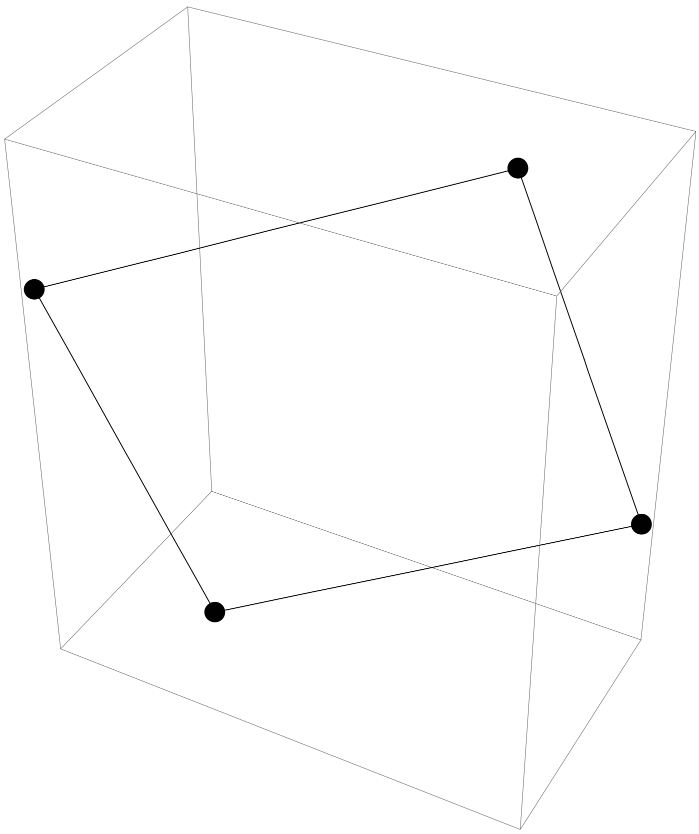
```
{Norm[pA1 - pB], Norm[pC - pD]}
```

$\left\{9\sqrt{2}, 9\sqrt{2}\right\}$

```
{Norm[pA1 - pD], Norm[pC - pB]}
```

$\left\{9\sqrt{2}, 9\sqrt{2}\right\}$

```
Show[Graphics3D[{PointSize[0.03], Point[pA1], Point[pB], Point[pC],
   Point[pD], Line[{pA1, pB, pC, pD, pA1}]}], AspectRatio -> Automatic]
```



- **d**

```
gS[t_] := pA1 + 1 / 2 (pC - pA1) + t Cross[pB - pA1, pD - pA1];
gS[t]
```

{9 + 72 t, 6 - 144 t, 4 - 18 t}

```
Vol[t_] := Det[{pB - pA1, pD - pA1, gS[t] - pA1}] / 3;
Vol[t]
```

8748 t

```
solv5 = Solve[Sqrt[Vol[t]^2] == 1944, {t}] // Flatten
```

$\left\{t \to -\frac{2}{9},\ t \to \frac{2}{9}\right\}$

```
N[%]
```

{t → -0.222222, t → 0.222222}

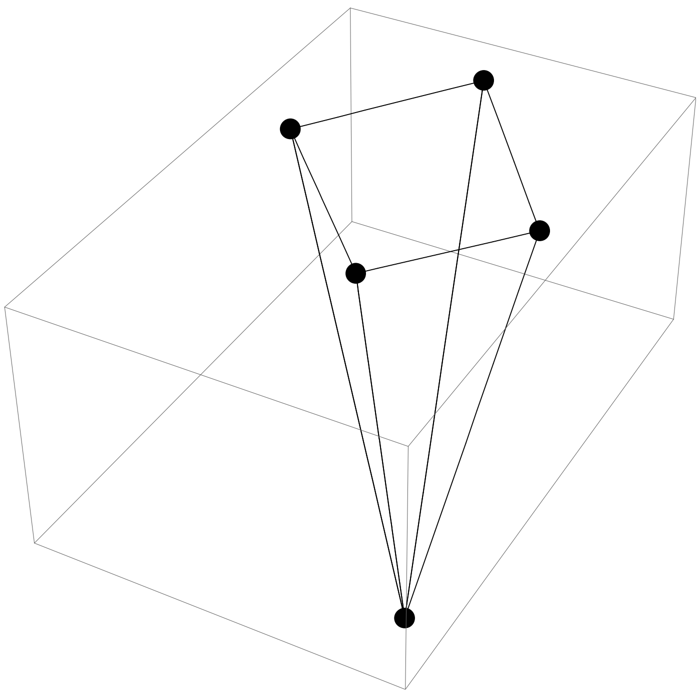
```
pS1 = gS[t] /. solv5[[1]]
```

{-7, 38, 8}

```
pS2 = gS[t] /. solv5[[2]]
```

{25, -26, 0}

```
Show[Graphics3D[
  {PointSize[0.03], Point[pA1], Point[pB], Point[pC], Point[pD], Point[pS2], Line[
    {pA1, pB, pS2, pB, pC, pS2, pC, pD, pS2, pD, pA1, pS2}]}], AspectRatio -> Automatic]
```



```
(* Kontrolle *)  Det[{pB - pA1, pD - pA1, pS2 - pA1}] / 3
```

1944

3.2 Rechter Winkel bei C (Variante problematisch)

```
(* Variante problematisch, da ABCD Rechteck! *) Remove["Global`*"]
```

- **a**

```
pA[y_] := {16, y, 0}; pB = {13, 7, 12}; pC = {2, 2, 8};
{pA[y], pB, pC}
```

{{16, y, 0}, {13, 7, 12}, {2, 2, 8}}

```
solve4 = Solve[(pA[y] - pC).(pB - pC) == 0, {y}] // Flatten
```

$\left\{y \to -\frac{112}{5}\right\}$

```
N[%]
```

{y → -22.4}

```
pA1 = pA[y] /. solve4
```

$\left\{16, -\frac{112}{5}, 0\right\}$

```
N[%]
```

{16., -22.4, 0.}

b

```
{Norm[pB - pA1], Norm[pC - pB], Norm[pC - pA1]}
```

$\left\{\frac{9\sqrt{314}}{5}, 9\sqrt{2}, \frac{18\sqrt{66}}{5}\right\}$

Nicht gleichschenklig

c

```
pD = pA1 + (-1) (pC - pB)
```

$\left\{27, -\frac{87}{5}, 4\right\}$

```
N[%]
```

{27., -17.4, 4.}

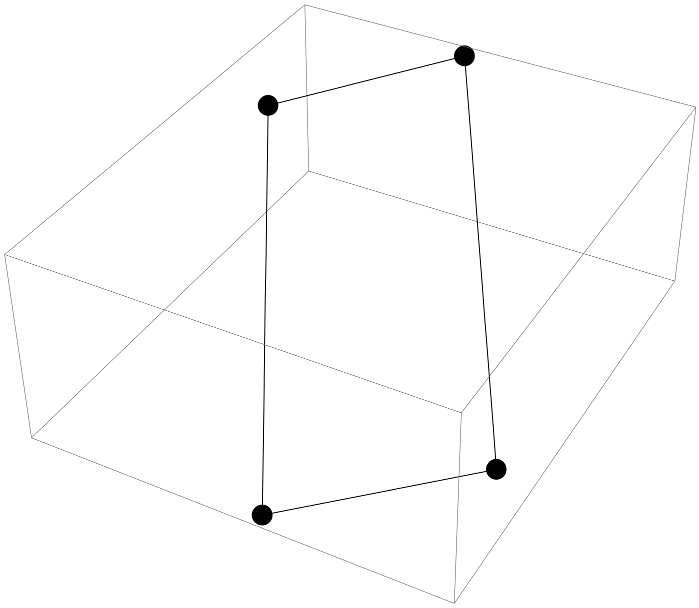
```
{Norm[pA1 - pB], Norm[pC - pD]}
```

$\left\{\frac{9\sqrt{314}}{5}, \frac{9\sqrt{314}}{5}\right\}$

```
{Norm[pA1 - pD], Norm[pC - pB]}
```

$\left\{9\sqrt{2}, 9\sqrt{2}\right\}$

```
Show[Graphics3D[{PointSize[0.03], Point[pA1], Point[pB], Point[pC],
   Point[pD], Line[{pA1, pC, pB, pD, pA1}]}], AspectRatio -> Automatic]
```



d

```
gS[t_] := pA1 + 1 / 2 (pB - pA1) + t Cross[pC - pA1, pD - pA1];
gS[t]
```

$$\left\{\frac{29}{2}+\frac{288\,t}{5},\ -\frac{77}{10}+144\,t,\ 6-\frac{1692\,t}{5}\right\}$$

```
Vol[t_] := Det[{pC - pA1, pD - pA1, gS[t] - pA1}] / 3;
Vol[t]
```

$$\frac{1\,154\,736\,t}{25}$$

```
solv5 = Solve[Sqrt[Vol[t]^2] == 1944, {t}] // Flatten
```

$$\left\{t\to -\frac{25}{594},\ t\to \frac{25}{594}\right\}$$

```
N[%]
```

{t → -0.0420875, t → 0.0420875}

```
pS1 = gS[t] /. solv5[[1]]
```

$$\left\{\frac{797}{66},\ -\frac{4541}{330},\ \frac{668}{33}\right\}$$

```
N[%]
```

{12.0758, -13.7606, 20.2424}

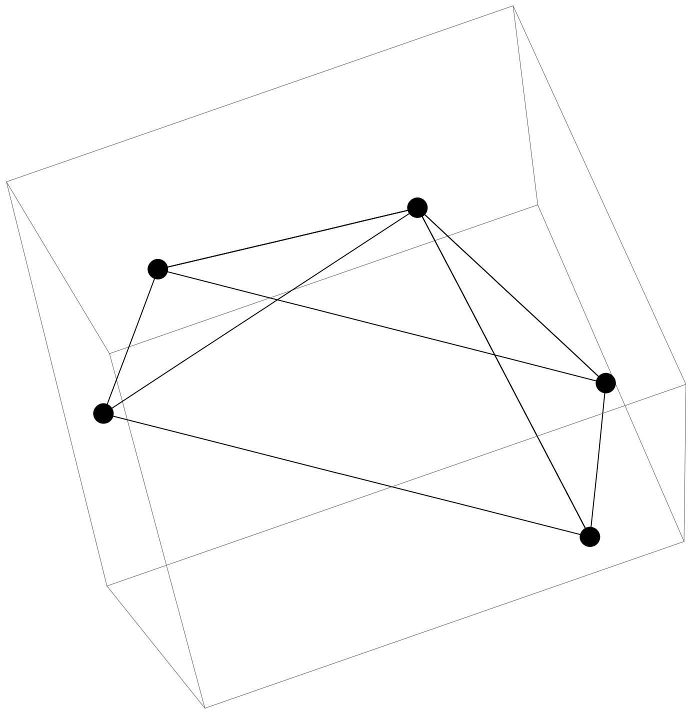
```
pS2 = gS[t] /. solv5[[2]]
```

$$\left\{\frac{1117}{66},\ -\frac{541}{330},\ -\frac{272}{33}\right\}$$

```
N[%]
```

{16.9242, -1.63939, -8.24242}

```
Show[Graphics3D[
  {PointSize[0.03], Point[pA1], Point[pB], Point[pC], Point[pD], Point[pS2], Line[
    {pA1, pC, pS2, pC, pB, pS2, pB, pD, pS2, pD, pA1, pS2}]}], AspectRatio -> Automatic]
```



```
(* Kontrolle *)  Det[{pC - pA1, pD - pA1, pS2 - pA1}] / 3
```

1944

3.3 Rechter Winkel bei A: Keine reelle Lösung!

```
Remove["Global`*"]
```

- **a**

```
pA[y_] := {16, y, 0}; pB = {13, 7, 12}; pC = {2, 2, 8};
{pA[y], pB, pC}
```

{{16, y, 0}, {13, 7, 12}, {2, 2, 8}}

```
solve4 = Solve[(pC - pA[y]).(pB - pA[y]) == 0, {y}] // Flatten
```

$\left\{y \to \frac{1}{2}\left(9 - i\sqrt{527}\right),\ y \to \frac{1}{2}\left(9 + i\sqrt{527}\right)\right\}$

```
N[%]
```

{y → 4.5 − 11.4782 i, y → 4.5 + 11.4782 i}

```
pA1 = pA[y] /. solve4
```

$\left\{16,\ \frac{1}{2}\left(9 - i\sqrt{527}\right),\ 0\right\}$

```
N[%]
```

{16., 4.5 − 11.4782 i, 0.}

Keine reelle Lösung.

4

```
Remove["Global`*"]
```

```
P0 = {0, 0}; P1 = {5, 0}; P2 = {4, 1}; P3 = {3.5, 2}; P4 = {2, 6}; P5 = {-1, 5};
A[p1_, p2_] := p1[[1]] p2[[2]] - p2[[1]] p1[[2]];
```

a

```
A[P1, P2] / 2 // N
```

```
2.5
```

```
A[P2, P3] / 2
```

```
2.25
```

```
A[P3, P4] / 2
```

```
8.5
```

```
A[P4, P5] / 2
```

```
8
```

```
(A[P1, P2] + A[P2, P3] + A[P3, P4] + A[P4, P5]) / 2
```

```
21.25
```

b

```
Dr[ϕ_] := {{Cos[ϕ], -Sin[ϕ]}, {Sin[ϕ], Cos[ϕ]}} ; Dr[ϕ] // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}[\phi] & -\mathrm{Sin}[\phi] \\ \mathrm{Sin}[\phi] & \mathrm{Cos}[\phi] \end{pmatrix}$$

```
Dr[34.7 Degree] // MatrixForm
```

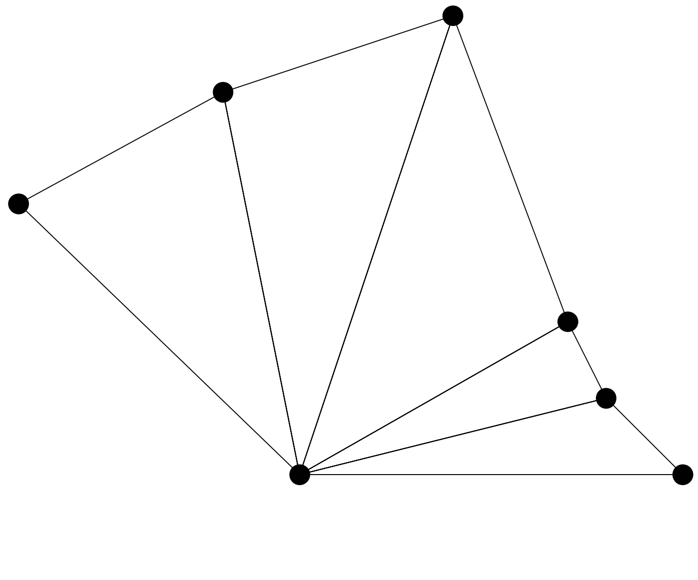
$$\begin{pmatrix} 0.822144 & -0.56928 \\ 0.56928 & 0.822144 \end{pmatrix}$$

c

```
P6 = Dr[34.7 Degree].P5
```

```
{-3.66854, 3.54144}
```

```
Show[Graphics[{PointSize[0.03], Point[P0],
   Point[P1], Point[P2], Point[P3], Point[P4], Point[P5], Point[P6],
   Line[{P0, P1, P2, P0, P2, P3, P0, P3, P4, P0, P4, P5, P0, P5, P6, P0}]}]]
```



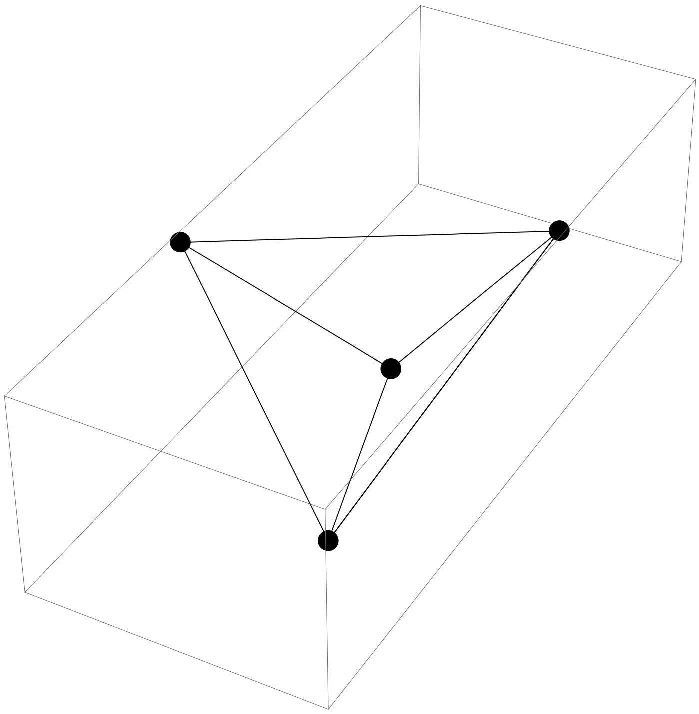
5

```
Remove["Global`*"]
```

```
pA = {2, 1, 0}; pB = {3, 8, 0}; pC = {-1, -3, 5}; pD = {6, -9, 4};
```

a

```
Show[Graphics3D[{PointSize[0.03], Point[pA], Point[pB], Point[pC], Point[pD],
   Line[{pA, pB, pC, pA, pD, pB, pD, pC}]}]]
```



```
Vol = Det[{pB - pA, pC - pA, pD - pA}] / 6
```

43

a

```
InhFlaeche = 1 / 2 (Norm[Cross[pB - pA, pC - pA]] + Norm[Cross[pA - pD, pB - pD]] +
    Norm[Cross[pB - pD, pC - pD]] + Norm[Cross[pC - pD, pA - pD]])
```

$$\frac{1}{2}\left(9\sqrt{19}+2\sqrt{561}+2\sqrt{1074}+3\sqrt{1427}\right)$$

```
N[%]
```

132.736

5

```
Remove["Global`*"]
```

```
M1 = {{-11, 8}, {8, 1}}; M1 // MatrixForm
```

$$\begin{pmatrix} -11 & 8 \\ 8 & 1 \end{pmatrix}$$

```
M2 = {{2, -6}, {-6, -7}}; M2 // MatrixForm
```

$$\begin{pmatrix} 2 & -6 \\ -6 & -7 \end{pmatrix}$$

- **a**

```
Eigenvalues[M1]
```

{-15, 5}

- **b**

```
Eigenvectors[M1]
```

{{-2, 1}, {1, 2}}

```
Eigenvectors[M1][[1]] // Norm
```

$\sqrt{5}$

```
nEV1[k_] := Eigenvectors[M1][[k]] / Norm[Eigenvectors[M1][[k]]] // N
nEV1[1]
```

{-0.894427, 0.447214}

```
nEV1[2]
```

{0.447214, 0.894427}

- **c**

```
Eigenvalues[M2]
```

{-10, 5}

- **d**

```
nEV2[k_] := Eigenvectors[M2][[k]] / Norm[Eigenvectors[M2][[k]]] // N
nEV2[1]
```

{0.447214, 0.894427}

```
nEV2[2]
```

{-0.894427, 0.447214}

- **e**

Gleiche Eigenvektoren, verschiedene Eigenwerte

- **f**

```
M1.M2 // MatrixForm
```

$$\begin{pmatrix} -70 & 10 \\ 10 & -55 \end{pmatrix}$$

```
M2.M1 // MatrixForm
```

$$\begin{pmatrix} -70 & 10 \\ 10 & -55 \end{pmatrix}$$

Kommutativ

- **g**

```
Eigenvalues[M1.M2]
```

{-75, -50}

```
{5, -10} {-15, 5}
```

{-75, -50}

Produkte der EW von M1 und M2

- **h**

```
nEV3[k_] := Eigenvectors[M1.M2][[k]] / Norm[Eigenvectors[M1.M2][[k]]] // N
```

```
nEV3[1]
```

```
{-0.894427, 0.447214}
```

```
nEV3[2]
```

```
{0.447214, 0.894427}
```

EV von M1 und M2

- **i**

```
Eigenvalues[M1.M2.M1]
```

```
{1125, -250}
```

```
{-15, 5} {5, -10} {-15, 5}
```

```
{1125, -250}
```

Produkte der EW von M1 und M2 und M1

- **j**

```
a = {2, 4}; b = {3, 2}; o = {0, 0};
FlProd[p1_, p2_] := p1[[1]] p2[[2]] - p2[[1]] p1[[2]];
```

```
a1 = M1.a
```

```
{10, 20}
```

```
b1 = M1.b
```

```
{-17, 26}
```

```
FlProd[a, b]
```

```
-8
```

```
FlProd[a1, b1]
```

```
600
```

```
Det[M1]
```

```
-75
```

```
Eigenvalues[M1][[1]] Eigenvalues[M1][[2]]
```

```
-75
```

```
FlProd[a, b] Det[M1] == FlProd[a1, b1]
```

```
True
```