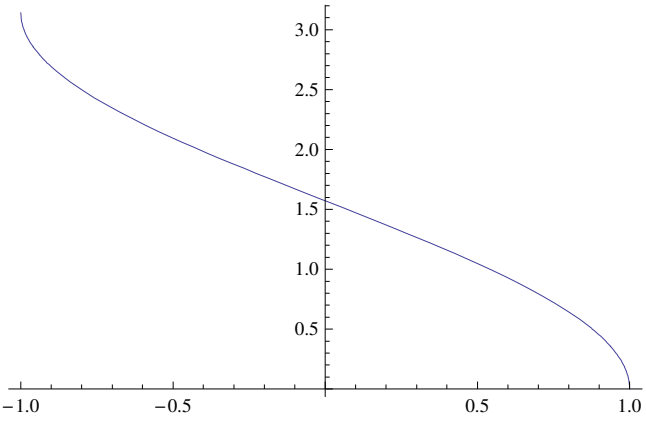
Lösungen

1

■ a

```
Plot[ArcCos[x], {x, -1, 1}]
```




```
Pi /Degree // N
```

```
180.
```

```
pO={0,0,0};p1={3,2,1};p2={-3,2,1};p3={3,-2,1};p4={3,2,-1};
winkelR[p1_,p2_]:= ArcCos[p1.p2/Norm[p1]/Norm[p2]];
winkelN[p1_,p2_]:= ArcCos[p1.p2/Norm[p1]/Norm[p2]]//N;
winkelD[p1_,p2_]:= ArcCos[p1.p2/Norm[p1]/Norm[p2]]/Degree//N;
winkel[p1_,p2_]:= {winkelR[p1, p2],winkelN[p1, p2] " Rad",winkelD[p1, p2] " Grad",
(180-winkelD[p1, p2]) " Grad"}
```

```
winkel[p1, p2]
```

$$\left\{\text{ArcCos}\left[-\frac{2}{7}\right], 1.86055\ \text{Rad}, 106.602\ \text{Grad}, 73.3985\ \text{Grad}\right\}$$

```
winkel[p1, p3]
```

$$\left\{\text{ArcCos}\left[\frac{3}{7}\right], 1.12789\ \text{Rad}, 64.6231\ \text{Grad}, 115.377\ \text{Grad}\right\}$$

```
winkel[p1, p4]
```

$$\left\{\text{ArcCos}\left[\frac{6}{7}\right], 0.5411\ \text{Rad}, 31.0027\ \text{Grad}, 148.997\ \text{Grad}\right\}$$

```
winkel[p2, p3]
```

$$\left\{\text{ArcCos}\left[-\frac{6}{7}\right], 2.60049\ \text{Rad}, 148.997\ \text{Grad}, 31.0027\ \text{Grad}\right\}$$

```
winkel[p2, p4]
```

$$\left\{\text{ArcCos}\left[-\frac{3}{7}\right], 2.01371\ \text{Rad}, 115.377\ \text{Grad}, 64.6231\ \text{Grad}\right\}$$

```
winkel[p3, p4]
```

$$\left\{\text{ArcCos}\left[\frac{2}{7}\right],\ 1.28104\ \text{Rad},\ 73.3985\ \text{Grad},\ 106.602\ \text{Grad}\right\}$$

■ b

```
inhaltE[p1_, p2_] := Norm[Cross[2 p1, 2 p2]] / 2;
inhaltN[p1_, p2_] := inhaltE[p1, p2] // N;
inhalt[p1_, p2_] := {inhaltE[p1, p2] , " = ", inhaltN[p1, p2], inhaltN[p1, p2] / 4};
```

```
inhalt[p1, p2]
```

$$\left\{12\sqrt{5},\ =\ ,\ 26.8328,\ 6.7082\right\}$$

```
inhalt[p1, p3]
```

$$\left\{8\sqrt{10},\ =\ ,\ 25.2982,\ 6.32456\right\}$$

```
inhalt[p1, p4]
```

$$\left\{4\sqrt{13},\ =\ ,\ 14.4222,\ 3.60555\right\}$$

```
inhalt[p2, p3]
```

$$\left\{4\sqrt{13},\ =\ ,\ 14.4222,\ 3.60555\right\}$$

```
inhalt[p2, p4]
```

$$\left\{8\sqrt{10},\ =\ ,\ 25.2982,\ 6.32456\right\}$$

```
inhalt[p3, p4]
```

$$\left\{12\sqrt{5},\ =\ ,\ 26.8328,\ 6.7082\right\}$$

2

```
v = {1, 2, -1}; b1 = {2, 3, -1}; b2 = {2, 3, 1}; b3 = {2, -1, 2};
Solve[v == λ b1 + μ b2 + ν b3, {λ , μ, ν}]
```

$$\left\{\left\{\lambda \to \frac{11}{16},\ \mu \to -\frac{1}{16},\ \nu \to -\frac{1}{8}\right\}\right\}$$

```
N[%]
```

$$\{\{\lambda \to 0.6875,\ \mu \to -0.0625,\ \nu \to -0.125\}\}$$

3

```
Det[{b1, b2, b3}]
```

$$16$$

4

```
r1 = {1, 2, -1}; r2 = {2, 2, 1}; a1 = {3, 1, 1}; a2 = {1, -1, 4};
kuerzesterAbstand = Abs[Det[{r2 - r1, a1, a2}] / Norm[Cross[a1, a2]]]
```

$$\frac{1}{3\sqrt{2}}$$

```
% // N
```

```
0.235702
```

5

a

```
vec[x_, y_, z_] := {x, y, z};
{vec[x, y, z].a1 == 0, vec[x, y, z].a2 == 0, vec[x, y, z].r2 == 10}
```

```
{3 x + y + z == 0, x - y + 4 z == 0, 2 x + 2 y + z == 10}
```

```
solv =
 Solve[{vec[x, y, z].a1 == 0, vec[x, y, z].a2 == 0, vec[x, y, z].r2 == 10}, {x, y, z}] // Flatten
```

$$\left\{x \to -\frac{25}{8},\ y \to \frac{55}{8},\ z \to \frac{5}{2}\right\}$$

```
% // N
```

```
{{x → -3.125, y → 6.875, z → 2.5}}
```

b

```
vec[x_, y_, z_] := {x, y, z} /. solv;  vec[x, y, z]
```

$$\left\{-\frac{25}{8},\ \frac{55}{8},\ \frac{5}{2}\right\}$$

```
Norm[Cross[vec[x, y, z], r2]]
```

$$\frac{5\sqrt{\frac{601}{2}}}{4}$$

```
% // N
```

```
21.6687
```

c

```
winkel[vec[x, y, z], r2]
```

$$\left\{\text{ArcCos}\left[\frac{8\sqrt{2}}{27}\right],\ 1.13842\ \text{Rad},\ 65.2269\ \text{Grad},\ 114.773\ \text{Grad}\right\}$$

6

■ a

```
w = {{2}, {-1}, {2}}; w // MatrixForm
```

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

```
dreh[ϕ_] := {{Cos[ϕ], -Sin[ϕ], 0}, {Sin[ϕ], Cos[ϕ], 0}, {0, 0, 1}}; dreh[ϕ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\phi] & -\text{Sin}[\phi] & 0 \\ \text{Sin}[\phi] & \text{Cos}[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
dreh[Pi / 2].{{1}, {0}, {0}} // MatrixForm
```

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

```
Table[dreh[n 2 Pi / 5].w // MatrixForm, {n, 1, 5}] // Simplify
```

$$\left\{ \begin{pmatrix} \frac{1}{4}\left(-2+2\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right) \\ \frac{1}{4}\left(1-\sqrt{5}+2\sqrt{2\left(5+\sqrt{5}\right)}\right) \\ 2 \end{pmatrix}, \begin{pmatrix} \frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\left(1+\sqrt{5}\right)\right) \\ \frac{1}{4}\left(1+\sqrt{5}+2\sqrt{10-2\sqrt{5}}\right) \\ 2 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} \frac{1}{4}\left(-\sqrt{10-2\sqrt{5}}-2\left(1+\sqrt{5}\right)\right) \\ \frac{1}{4}\left(1+\sqrt{5}-2\sqrt{10-2\sqrt{5}}\right) \\ 2 \end{pmatrix}, \begin{pmatrix} \frac{1}{4}\left(-2+2\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right) \\ \frac{1}{4}\left(1-\sqrt{5}-2\sqrt{2\left(5+\sqrt{5}\right)}\right) \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\}$$

```
Table[dreh[n 2 Pi / 5].w // MatrixForm, {n, 1, 5}] // Simplify // ExpandAll
```

$$\left\{ \begin{pmatrix} -\frac{1}{2}+\frac{\sqrt{5}}{2}+\frac{1}{4}\sqrt{10+2\sqrt{5}} \\ \frac{1}{4}-\frac{\sqrt{5}}{4}+\frac{1}{2}\sqrt{10+2\sqrt{5}} \\ 2 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2}-\frac{\sqrt{5}}{2}+\frac{1}{4}\sqrt{10-2\sqrt{5}} \\ \frac{1}{4}+\frac{\sqrt{5}}{4}+\frac{1}{2}\sqrt{10-2\sqrt{5}} \\ 2 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} -\frac{1}{2}-\frac{\sqrt{5}}{2}-\frac{1}{4}\sqrt{10-2\sqrt{5}} \\ \frac{1}{4}+\frac{\sqrt{5}}{4}-\frac{1}{2}\sqrt{10-2\sqrt{5}} \\ 2 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2}+\frac{\sqrt{5}}{2}-\frac{1}{4}\sqrt{10+2\sqrt{5}} \\ \frac{1}{4}-\frac{\sqrt{5}}{4}-\frac{1}{2}\sqrt{10+2\sqrt{5}} \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\}$$

```
N[%]
```

$$\left\{ \begin{pmatrix} 1.56909 \\ 1.5931 \\ 2. \end{pmatrix}, \begin{pmatrix} -1.03025 \\ 1.98459 \\ 2. \end{pmatrix}, \begin{pmatrix} -2.20582 \\ -0.366554 \\ 2. \end{pmatrix}, \begin{pmatrix} -0.333023 \\ -2.21113 \\ 2. \end{pmatrix}, \begin{pmatrix} 2. \\ -1. \\ 2. \end{pmatrix} \right\}$$

b

```
(dreh[2 Pi / 5].w)
```

$$\left\{\left\{\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}+\frac{1}{2}\left(-1+\sqrt{5}\right)\right\},\ \left\{\frac{1}{4}\left(1-\sqrt{5}\right)+2\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\right\},\ \{2\}\right\}$$

```
inhalt = 5 Transpose[(dreh[2 Pi / 5].w)].(dreh[2 * 2 Pi / 5].w) / 2  // Simplify
```

$$\left\{\left\{\frac{5}{8}\left(11+5\sqrt{5}\right)\right\}\right\}$$

```
N[%]
```

```
{{13.8627}}
```