Lösungen

1

```
Remove["Global`*"]

r0 = {1, 2, -1};
a = {-2, 1, 1};
q = {3, 10, 14};
rq = q - r0
```

```
{2, 8, 15}
```

```
gq = Norm[Cross[a, rq]] / Norm[a]
```

$$\sqrt{\frac{1397}{6}}$$

```
N[%]
```

```
15.2589
```

2

```
Remove["Global`*"]

r0 = {1, 2, -1};
a = {2, 1, -1};
b = {1, -1, 2};
q = {3, 10, 14};
rq = q - r0
```

```
{2, 8, 15}
```

a

```
gq = -Det[{a, b, rq}] / Norm[Cross[a, b]]
```

$$\frac{83}{\sqrt{35}}$$

```
N[%]
```

```
14.0296
```

b

```
v[λ_, μ_] := r0 + λ a + μ b; v[λ, μ]
```

$\{1 + 2\,\lambda + \mu,\ 2 + \lambda - \mu,\ -1 - \lambda + 2\,\mu\}$

```
w[t_] := q + t Cross[a, b]; w[t]
```

$\{3 + t,\ 10 - 5\,t,\ 14 - 3\,t\}$

```
solv1 = Flatten[Solve[ v[λ, μ] == w[t], {λ, μ , t}]]
```

$\left\{\lambda \to \frac{6}{35},\ \mu \to \frac{141}{35},\ t \to \frac{83}{35}\right\}$

```
t0 = t /. solv1
```

$\frac{83}{35}$

```
w[t0]
```

$\left\{\frac{188}{35},\ -\frac{13}{7},\ \frac{241}{35}\right\}$

```
N[%]
```

$\{5.37143,\ -1.85714,\ 6.88571\}$

3

```
Remove["Global`*"]
```

```
M = {{1, -1}, {2, 1}}; M // MatrixForm
```

$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$

a

```
ew = Eigenvalues[M]
```

$\left\{1 + i\sqrt{2},\ 1 - i\sqrt{2}\right\}$

```
N[%]
```

$\{1. + 1.41421\,i,\ 1. - 1.41421\,i\}$

```
Abs[ew[[1]]] E^(I Arg[ew[[1]]])
```

$\sqrt{3}\ e^{i\,\text{ArcTan}\left[\sqrt{2}\right]}$

```
{N[Abs[ew[[1]]]], I N[Arg[ew[[1]]]]}
```

$\{1.73205,\ 0.955317\,i\}$

```
Abs[ew[[2]]] E^(I Arg[ew[[2]]])
```

$$\sqrt{3}\ e^{-i\ \text{ArcTan}\left[\sqrt{2}\right]}$$

```
{N[Abs[ew[[2]]]], E^(I N[Arg[ew[[2]]]])}
```

$$\{1.73205,\ 0.57735 - 0.816497\ i\}$$

```
ev = Eigenvectors[M] // ExpandAll
```

$$\left\{\left\{\frac{i}{\sqrt{2}},\ 1\right\},\ \left\{-\frac{i}{\sqrt{2}},\ 1\right\}\right\}$$

```
N[%]
```

$$\{\{0. + 0.707107\ i,\ 1.\},\ \{0. - 0.707107\ i,\ 1.\}\}$$

b

```
{ev[[1]] / Norm[ev[[1]]], ev[[2]] / Norm[ev[[2]]]}
```

$$\left\{\left\{\frac{i}{\sqrt{3}},\ \sqrt{\frac{2}{3}}\right\},\ \left\{-\frac{i}{\sqrt{3}},\ \sqrt{\frac{2}{3}}\right\}\right\}$$

```
N[%]
```

$$\{\{0. + 0.57735\ i,\ 0.816497\},\ \{0. - 0.57735\ i,\ 0.816497\}\}$$

```
Eigensystem[M]
```

$$\left\{\left\{1 + i\sqrt{2},\ 1 - i\sqrt{2}\right\},\ \left\{\left\{-\frac{1}{2} + \frac{1}{2}\left(1 + i\sqrt{2}\right),\ 1\right\},\ \left\{-\frac{1}{2} + \frac{1}{2}\left(1 - i\sqrt{2}\right),\ 1\right\}\right\}\right\}$$

c

```
X = Transpose[ev]
```

$$\left\{\left\{\frac{i}{\sqrt{2}},\ -\frac{i}{\sqrt{2}}\right\},\ \{1,\ 1\}\right\}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0. + 0.707107\ i & 0. - 0.707107\ i \\ 1. & 1. \end{pmatrix}$$

```
Inverse[X]
```

$$\left\{\left\{-\frac{i}{\sqrt{2}},\ \frac{1}{2}\right\},\ \left\{\frac{i}{\sqrt{2}},\ \frac{1}{2}\right\}\right\}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0. - 0.707107\ i & 0.5 \\ 0. + 0.707107\ i & 0.5 \end{pmatrix}$$

```
dM = {{ew[[1]], 0}, {0, ew[[2]]}}
```

$$\left\{\left\{1 + i\sqrt{2},\ 0\right\},\ \left\{0,\ 1 - i\sqrt{2}\right\}\right\}$$

```
dM // N // MatrixForm
```

$$\begin{pmatrix} 1. + 1.41421\,i & 0. \\ 0. & 1. - 1.41421\,i \end{pmatrix}$$

```
X.dM.Inverse[X] // N // Chop // MatrixForm
```

$$\begin{pmatrix} 1. & -1. \\ 2. & 1. \end{pmatrix}$$

d

```
Eigenvalues[dM] == Eigenvalues[M]
```

```
True
```

e

```
Eigenvectors[dM]
```

```
{{1, 0}, {0, 1}}
```

```
Eigenvectors[dM] == Eigenvectors[M]
```

```
False
```

f Komplex

```
g1[t_] := t ev[[1]]; g1[t] // N
```

```
{(0. + 0.707107 i) t, t}
```

```
g2[t_] := t ev[[2]]; g2[t] // N
```

```
{(0. - 0.707107 i) t, t}
```

g

```
(M. g1[t] // Simplify // ExpandAll) / (1 + I Sqrt[2]) // Simplify // N // Chop
```

```
{0.707107 i t, t}
```

```
(M. g2[t] // Simplify // ExpandAll) / (1 - I Sqrt[2]) // Simplify // N // Chop
```

```
{-0.707107 i t, t}
```

h

```
(dM. g1[t] // N // Simplify // ExpandAll) / (1.` - 1.4142135623730951` i) // Chop
```

```
{(-0.666667 - 0.235702 i) t, 1. t}
```

```
(dM. g2[t] // N // Simplify // ExpandAll) / (1.` - 1.4142135623730951` i) // Chop
```

```
{(0.666667 + 0.235702 i) t, 1. t}
```

Keinde Deckung

i

```
M.M // MatrixForm
```

$$\begin{pmatrix} -1 & -2 \\ 4 & -1 \end{pmatrix}$$

```
ew[[2]]
```

$$1 - i\sqrt{2}$$

```
dM = {{ew[[1]], 0}, {0, ew[[2]]}};
X.(dM^2).Inverse[X] // N // Chop // MatrixForm
```

$$\begin{pmatrix} -1. & -2. \\ 4. & -1. \end{pmatrix}$$

```
X.(dM^2).Inverse[X] // Simplify // MatrixForm
```

$$\begin{pmatrix} -1 & -2 \\ 4 & -1 \end{pmatrix}$$

j

```
dM = {{ew[[1]], 0}, {0, ew[[2]]}};
X.(dM^50).Inverse[X] // N // Chop // MatrixForm
```

$$\begin{pmatrix} -6.78623 \times 10^{11} & 3.58726 \times 10^{11} \\ -7.17452 \times 10^{11} & -6.78623 \times 10^{11} \end{pmatrix}$$

```
X.(dM^50).Inverse[X] // Simplify // MatrixForm
```

$$\begin{pmatrix} -678623127841 & 358725966478 \\ -717451932956 & -678623127841 \end{pmatrix}$$

4

```
Remove["Global`*"]
```

```
normVec[menge_] := Table[ menge[[k]] / Norm[menge[[k]]], {k, 1, Length[menge]}]
```

a

```
a1 = {1, 1, 0} ; a2 = {0, 1, 1} ;  a3 = {1, 0, -1} ;
```

```
A = Transpose[{a1, a2, a3}]; A // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

b

```
Eigenvalues[A]
```

$$\left\{\frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right), 0\right\}$$

```
N[%]
```

```
{1.61803, -0.618034, 0.}
```

c

```
Eigenvectors[A]
```

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 1+\frac{1}{2}\left(1+\sqrt{5}\right), 1\right\}, \left\{\frac{1}{2}\left(1-\sqrt{5}\right), 1+\frac{1}{2}\left(1-\sqrt{5}\right), 1\right\}, \{-1, 1, 1\}\right\}$$

```
menge = N[%]
```

```
{{1.61803, 2.61803, 1.}, {-0.618034, 0.381966, 1.}, {-1., 1., 1.}}
```

```
normVec[menge]
```

```
{{0.5, 0.809017, 0.309017}, {-0.5, 0.309017, 0.809017}, {-0.57735, 0.57735, 0.57735}}
```

d

```
I3 = IdentityMatrix[3]
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
A.I3 == A
```

```
True
```

```
Volumen = Det[A]
```

```
0
```

(Zu erwarten, da EW 0 existiert)

e

```
Det[A]
```

```
0
```

```
Det[A]^-1
```

```
Power::infy : Infinite expression 1/0 encountered. Mehr…
```

```
ComplexInfinity
```

f

```
Det[A] == Volumen
```

```
True
```

g

```
p = Product[Eigenvalues[A][[k]], {k, 1, Length[Eigenvalues[A]]}]
```

```
0
```

h

```
p == Det[A]
```

```
True
```

i

```
s = Sum[I3[[k]], {k, 1, Length[I3]}]
```

```
{1, 1, 1}
```

```
A.s
```

```
{2, 2, 0}
```

j

```
sA = Sum[A[[k]], {k, 1, Length[I3]}]
```

```
{2, 2, 0}
```

```
A.s == sA
```

```
True
```

5

```
Remove["Global`*"]
```

```
a = {-1, 3}; aS = {-3, -1};
```

```
v[t_] := t a; p = {10, -1};
```

a

```
{a, aS}
```

```
{{-1, 3}, {-3, -1}}
```

```
vS = Transpose[{a, aS}]; vS // MatrixForm
```

$$\begin{pmatrix} -1 & -3 \\ 3 & -1 \end{pmatrix}$$

```
dS = {{1, 0}, {0, -1}}; dS // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

```
S = vS.dS.Inverse[vS]; S // MatrixForm
```

$$\begin{pmatrix} -\frac{4}{5} & -\frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{pmatrix}$$

b

```
p1 = S.p
```

$$\left\{-\frac{37}{5}, -\frac{34}{5}\right\}$$

```
N[%]
```

```
{-7.4, -6.8}
```

c

```
α = 10 Degree; α // N
```

```
0.174533
```

```
α1 = -10 Degree
```

```
-10 °
```

```
d = {{Cos[α], -Sin[α]}, {Sin[α], Cos[α]}}; d // MatrixForm // N
```

$$\begin{pmatrix} 0.984808 & -0.173648 \\ 0.173648 & 0.984808 \end{pmatrix}$$

```
d1 = {{Cos[-α], -Sin[-α]}, {Sin[-α], Cos[-α]}}
```

$$\{\{\text{Cos}[10\,^\circ], \text{Sin}[10\,^\circ]\}, \{-\text{Sin}[10\,^\circ], \text{Cos}[10\,^\circ]\}\}$$

```
p2 = d.p1
```

$$\left\{-\frac{37}{5}\,\text{Cos}[10\,^\circ] + \frac{34}{5}\,\text{Sin}[10\,^\circ], -\frac{34}{5}\,\text{Cos}[10\,^\circ] - \frac{37}{5}\,\text{Sin}[10\,^\circ]\right\}$$

```
N[%]
```

$$\{-6.10677, -7.98169\}$$

d

```
p3 = S.p2
```

$$\left\{-\frac{3}{5}\left(-\frac{34}{5}\,\text{Cos}[10\,^\circ] - \frac{37}{5}\,\text{Sin}[10\,^\circ]\right) - \frac{4}{5}\left(-\frac{37}{5}\,\text{Cos}[10\,^\circ] + \frac{34}{5}\,\text{Sin}[10\,^\circ]\right),\right.$$
$$\left.\frac{4}{5}\left(-\frac{34}{5}\,\text{Cos}[10\,^\circ] - \frac{37}{5}\,\text{Sin}[10\,^\circ]\right) - \frac{3}{5}\left(-\frac{37}{5}\,\text{Cos}[10\,^\circ] + \frac{34}{5}\,\text{Sin}[10\,^\circ]\right)\right\}$$

```
N[%]
```

$$\{9.67443, -2.72129\}$$

e

```
p4 = d1.p
```

$$\{10\,\text{Cos}[10\,^\circ] - \text{Sin}[10\,^\circ], -\text{Cos}[10\,^\circ] - 10\,\text{Sin}[10\,^\circ]\}$$

```
N[%]
```

$$\{9.67443, -2.72129\}$$

g

```
(p3 // N) == (p4 // N)
```

```
True
```

6

```
Remove["Global`*"]
```

```
a = {2, -1, -1}; b = {1, 1, -3};
```

```
v[λ_, μ_] := λ a + μ b; p0 = {10, 5, -3}; pPr = {10, 10, 20};
```

a

```
vS = Transpose[{a, b, Cross[a, b]}]; vS // MatrixForm
```

$$\begin{pmatrix} 2 & 1 & 4 \\ -1 & 1 & 5 \\ -1 & -3 & 3 \end{pmatrix}$$

```
dS = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; dS // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
S = vS.dS.Inverse[vS]; S // MatrixForm
```

$$\begin{pmatrix} \frac{9}{25} & -\frac{4}{5} & -\frac{12}{25} \\ -\frac{4}{5} & 0 & -\frac{3}{5} \\ -\frac{12}{25} & -\frac{3}{5} & \frac{16}{25} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.36 & -0.8 & -0.48 \\ -0.8 & 0. & -0.6 \\ -0.48 & -0.6 & 0.64 \end{pmatrix}$$

b

```
p1 = S.p0
```

$$\left\{\frac{26}{25}, -\frac{31}{5}, -\frac{243}{25}\right\}$$

```
N[%]
```

```
{1.04, -6.2, -9.72}
```

c

```
vPr = Transpose[{a, b, Cross[a, b]}]; vPr // MatrixForm
```

$$\begin{pmatrix} 2 & 1 & 4 \\ -1 & 1 & 5 \\ -1 & -3 & 3 \end{pmatrix}$$

```
dPr = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}; dPr // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Pr = vPr.dPr.Inverse[vPr]; Pr // MatrixForm
```

$$\begin{pmatrix} \frac{17}{25} & -\frac{2}{5} & -\frac{6}{25} \\ -\frac{2}{5} & \frac{1}{2} & -\frac{3}{10} \\ -\frac{6}{25} & -\frac{3}{10} & \frac{41}{50} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.68 & -0.4 & -0.24 \\ -0.4 & 0.5 & -0.3 \\ -0.24 & -0.3 & 0.82 \end{pmatrix}$$

d

```
Q = Pr.pPr
```

```
{-2, -5, 11}
```

7

```
Remove["Global`*"]
```

```
M.(Id - X).Inverse[M] + M - A.M == A.Transpose[M] - 2 M
```

```
M - A.M + M.(Id - X).Inverse[M] == -2 M + A.Transpose[M]
```

```
Id - M.X.Inverse[M] + M - A.M == A.Transpose[M] - 2 M
```

```
Id + M - A.M - M.X.Inverse[M] == -2 M + A.Transpose[M]
```

```
Id - A.Transpose[M] + 3 M - A.M == M.X.Inverse[M]
```

```
Id + 3 M - A.M - A.Transpose[M] == M.X.Inverse[M]
```

```
X == Inverse[M].(Id - A.Transpose[M] + 3 M - A.M).M
```

```
X == Inverse[M].(Id + 3 M - A.M - A.Transpose[M]).M
```

```
X == Id - Inverse[M].A.Transpose[M].M + 3 M - Inverse[M].A.M.M
```

```
X == Id + 3 M - Inverse[M].A.M.M - Inverse[M].A.Transpose[M].M
```