Lösungen

1

```
Remove["Global`*"]
```

a

```
OP1 = {2, 0, 6};
a = {3, 1, 4};
b = {1, 0, 0};
c = Cross[a, b]
```

$$\{0, 4, -1\}$$

```
d = Cross[a, c]
```

$$\{-17, 3, 12\}$$

```
ea = a / Norm[a]
```

$$\left\{\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 2\sqrt{\frac{2}{13}}\right\}$$

```
ea // N
```

$$\{0.588348, 0.196116, 0.784465\}$$

```
ec = c / Norm[c]
```

$$\left\{0, \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right\}$$

```
ec // N
```

$$\{0., 0.970143, -0.242536\}$$

```
ed = d / Norm[d]
```

$$\left\{-\sqrt{\frac{17}{26}}, \frac{3}{\sqrt{442}}, 6\sqrt{\frac{2}{221}}\right\}$$

```
ed // N
```

$$\{-0.808608, 0.142695, 0.570782\}$$

b

```
M = Transpose[{ea, ec, ed}]; M // MatrixForm
```

$$\begin{pmatrix} \frac{3}{\sqrt{26}} & 0 & -\sqrt{\frac{17}{26}} \\ \frac{1}{\sqrt{26}} & \frac{4}{\sqrt{17}} & \frac{3}{\sqrt{442}} \\ 2\sqrt{\frac{2}{13}} & -\frac{1}{\sqrt{17}} & 6\sqrt{\frac{2}{221}} \end{pmatrix}$$

```
M // N // MatrixForm
```

$$\begin{pmatrix} 0.588348 & 0. & -0.808608 \\ 0.196116 & 0.970143 & 0.142695 \\ 0.784465 & -0.242536 & 0.570782 \end{pmatrix}$$

c

```
Minv = Inverse[M]
```

$$\left\{\left\{\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, 2\sqrt{\frac{2}{13}}\right\}, \left\{0, \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}}\right\}, \left\{-\sqrt{\frac{17}{26}}, \frac{3}{\sqrt{442}}, 6\sqrt{\frac{2}{221}}\right\}\right\}$$

```
Minv // MatrixForm
```

$$\begin{pmatrix} \frac{3}{\sqrt{26}} & \frac{1}{\sqrt{26}} & 2\sqrt{\frac{2}{13}} \\ 0 & \frac{4}{\sqrt{17}} & -\frac{1}{\sqrt{17}} \\ -\sqrt{\frac{17}{26}} & \frac{3}{\sqrt{442}} & 6\sqrt{\frac{2}{221}} \end{pmatrix}$$

```
Minv // N // MatrixForm
```

$$\begin{pmatrix} 0.588348 & 0.196116 & 0.784465 \\ 0. & 0.970143 & -0.242536 \\ -0.808608 & 0.142695 & 0.570782 \end{pmatrix}$$

```
OP1s = Minv.OP1
```

$$\left\{15\sqrt{\frac{2}{13}}, -\frac{6}{\sqrt{17}}, 36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right\}$$

```
OP1s // N
```

```
{5.88348, -1.45521, 1.80748}
```

d

```
Dreh[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}}
```

Dreh[2 Pi / 3] // MatrixForm

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Dreh[2 Pi / 3] // N // MatrixForm

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & -0.5 & -0.866025 \\ 0. & 0.866025 & -0.5 \end{pmatrix}$$

Dreh[4 Pi / 3] // MatrixForm

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Dreh[4 Pi / 3] // N // MatrixForm

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & -0.5 & 0.866025 \\ 0. & -0.866025 & -0.5 \end{pmatrix}$$

OP2s = Dreh[2 Pi / 3].OP1s

$$\left\{15\sqrt{\frac{2}{13}},\ \frac{3}{\sqrt{17}} - \frac{1}{2}\sqrt{3}\left(36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right),\ -3\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-36\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right\}$$

OP2s // N

{5.88348, −0.837713, −2.16399}

OP3s = Dreh[4 Pi / 3].OP1s

$$\left\{15\sqrt{\frac{2}{13}},\ \frac{3}{\sqrt{17}} + \frac{1}{2}\sqrt{3}\left(36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right),\ 3\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-36\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right\}$$

OP3s // N

{5.88348, 2.29293, 0.356514}

OP3as = Dreh[4 Pi / 3].OP2s

$$\left\{15\sqrt{\frac{2}{13}},\right.$$
$$\frac{1}{2}\left(-\frac{3}{\sqrt{17}} + \frac{1}{2}\sqrt{3}\left(36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right) + \frac{1}{2}\sqrt{3}\left(-3\sqrt{\frac{3}{17}} + \frac{1}{2}\left(-36\sqrt{\frac{2}{221}} + \sqrt{\frac{34}{13}}\right)\right),$$
$$\left.\frac{1}{2}\left(3\sqrt{\frac{3}{17}} + \frac{1}{2}\left(36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right) - \frac{1}{2}\sqrt{3}\left(\frac{3}{\sqrt{17}} - \frac{1}{2}\sqrt{3}\left(36\sqrt{\frac{2}{221}} - \sqrt{\frac{34}{13}}\right)\right)\right\}$$

OP3as // N

{5.88348, −1.45521, 1.80748}

e

```
OP2 = M.OP2s; OP2 // N
```

```
{5.21136, 0.0323537, 3.58339}
```

```
OP3 = M.OP3s; OP3 // N
```

```
{3.17326, 3.42918, 4.26276}
```

```
OP3a = M.OP3as; OP3a // N
```

```
{2., 0., 6.}
```

f

```
V = Det[{OP1, OP2, OP3}] / 6
```

$$\frac{525\sqrt{\frac{3}{26}}}{13}$$

```
V // N
```

```
13.718
```

```
Va = Det[{OP1, OP2, OP3a}] / 6
```

```
0
```

```
Va // N
```

```
0.
```

2

```
Remove["Global`*"]
```

a

```
a = {3, 1, 2}; b = {-1, -1, 0}; c = {1, -2, -2};
```

```
M = Transpose[{a, b, c}]; M // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & 1 \\ 1 & -1 & -2 \\ 2 & 0 & -2 \end{pmatrix}$$

```
Dl = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}; Dl // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
A = M.Dl.Inverse[M]; A // MatrixForm
```

$$\begin{pmatrix} \frac{4}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{2}{5} \\ \frac{2}{5} & -\frac{2}{5} & \frac{3}{5} \end{pmatrix}$$

```
A // N // MatrixForm
```

$$\begin{pmatrix} 0.8 & 0.2 & 0.2 \\ 0.4 & 0.6 & -0.4 \\ 0.4 & -0.4 & 0.6 \end{pmatrix}$$

b

```
OA = {1, 0, 0}; OB = {0, 1, 0}; OC = {0, 0, 1};
```

```
A.OA
```

$$\left\{\frac{4}{5}, \frac{2}{5}, \frac{2}{5}\right\}$$

```
N[%]
```

```
{0.8, 0.4, 0.4}
```

```
A.OB
```

$$\left\{\frac{1}{5}, \frac{3}{5}, -\frac{2}{5}\right\}$$

```
N[%]
```

```
{0.2, 0.6, -0.4}
```

```
A.OC
```

$$\left\{\frac{1}{5}, -\frac{2}{5}, \frac{3}{5}\right\}$$

```
N[%]
```

```
{0.2, -0.4, 0.6}
```

c

```
Norm[Cross[OB - OA, OC - OA]] / 2
```

$$\frac{\sqrt{3}}{2}$$

```
N[%]
```

```
0.866025
```

```
Norm[Cross[A.OB - A.OA, A.OC - A.OA]] / 2
```

$$\frac{3\sqrt{3}}{10}$$

```
N[%]
```

```
0.519615
```

3

```
Remove["Global`*"]
```

a

```
{{8, 1, -2}, {4, 5, -4}, {1, -1, 5}} // MatrixForm
```

$$\begin{pmatrix} 8 & 1 & -2 \\ 4 & 5 & -4 \\ 1 & -1 & 5 \end{pmatrix}$$

```
M3 = 1 / 3 {{8, 1, -2}, {4, 5, -4}, {1, -1, 5}}; M3 // MatrixForm
```

$$\begin{pmatrix} \frac{8}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{4}{3} & \frac{5}{3} & -\frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{5}{3} \end{pmatrix}$$

```
EW = Eigenvalues[M3]
```

```
{3, 2, 1}
```

```
EV = Eigenvectors[M3]
```

```
{{1, 1, 0}, {1, 0, 1}, {0, 2, 1}}
```

```
EV[[1]] / Norm[EV[[1]]]
```

$$\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$$

```
N[%]
```

```
{0.707107, 0.707107, 0.}
```

```
EV[[2]] / Norm[EV[[2]]]
```

$$\left\{\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}$$

```
N[%]
```

```
{0.707107, 0., 0.707107}
```

```
EV[[3]] / Norm[EV[[3]]]
```

$$\left\{0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}$$

```
N[%]
```

```
{0., 0.894427, 0.447214}
```

```
{{λ, 0, 0}, {0, λ, 0}, {0, 0, λ}} // MatrixForm
```

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

```
Id[λ_] := {{λ, 0, 0}, {0, λ, 0}, {0, 0, λ}}
```

b

```
p[λ_] := Det[M3 - Id[λ]]; p[λ] == 0
```

$$6 - 11\,\lambda + 6\,\lambda^2 - \lambda^3 == 0$$

```
p[λ] u // Expand
```

$$6\,u - 11\,u\,\lambda + 6\,u\,\lambda^2 - u\,\lambda^3$$

```
(p[λ] u // Expand) /. {(λ^3) → M3.M3.M3, (λ^2) → M3.M3, λ → M3, u -> Id[1]} // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
exp = (u (p[λ] - 6) / (-6 λ) // Expand) /. {u λ → λ, u λ^2 → λ^2}
```

$$\frac{11\,u}{6} - \lambda + \frac{\lambda^2}{6}$$

```
exp
```

$$\frac{11\,u}{6} - \lambda + \frac{\lambda^2}{6}$$

```
(exp /. {u -> Id[1], (λ^2) → M3.M3, λ → M3}) // MatrixForm
```

$$\begin{pmatrix} \frac{7}{18} & -\frac{1}{18} & \frac{1}{9} \\ -\frac{4}{9} & \frac{7}{9} & \frac{4}{9} \\ -\frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{pmatrix}$$

```
res = (exp /. {(λ^2) → M3.M3, λ → M3, u -> Id[1]}); res // MatrixForm
```

$$\begin{pmatrix} \frac{7}{18} & -\frac{1}{18} & \frac{1}{9} \\ -\frac{4}{9} & \frac{7}{9} & \frac{4}{9} \\ -\frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{pmatrix}$$

```
Inverse[M3] // MatrixForm
```

$$\begin{pmatrix} \frac{7}{18} & -\frac{1}{18} & \frac{1}{9} \\ -\frac{4}{9} & \frac{7}{9} & \frac{4}{9} \\ -\frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{pmatrix}$$

```
res == Inverse[M3]
```

```
True
```

```
Inverse[M3] == 1 / 6 M3.M3 - M3 + 11 / 6 Id[1]
```

```
True
```

4

```
Remove["Global`*"]
```

a

```
M4 = {{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}}; M4 // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(M4.M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(M4.M4.M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

b

```
(M4.M4).(M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(M4).(M4.M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(M4).(M4.M4.M4.M4) // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

5

```
Remove["Global`*"]
```

a

```
M5 = {{1, 2, 3, 4, 5} + {1, 1, 2, 1, 2},
{3, 2, 1, 5, 4},
{7, 2, -3, 7, 2},
{1, 1, 2, 1, 2} - {1, 2, 3, 4, 5}}; M5 // MatrixForm
```

$$\begin{pmatrix} 2 & 3 & 5 & 5 & 7 \\ 3 & 2 & 1 & 5 & 4 \\ 7 & 2 & -3 & 7 & 2 \\ 0 & -1 & -1 & -3 & -3 \end{pmatrix}$$

```
b1 = {4 + 2, 3, 1, 2 - 4}
```

```
{6, 3, 1, -2}
```

```
b2 = {4 + 2, 3, 0, 2 - 4}
```

```
{6, 3, 0, -2}
```

b

```
x = {x1, x2, x3, x4, x5};
```

```
Solve[M5.x == b2, x]
```

```
{}
```

```
Solve[M5.x == b1, x]
```

$$\left\{\left\{x1 \to -\frac{1}{4} + \frac{3\,x4}{4} + \frac{3\,x5}{4},\ x2 \to \frac{7}{4} - \frac{17\,x4}{4} - \frac{13\,x5}{4},\ x3 \to \frac{1}{4} + \frac{5\,x4}{4} + \frac{x5}{4}\right\}\right\}$$

c

Fall für b1: Dim Lösungsraum = 2

d

Rang = Ordnung - Dimension = 5 - 2 = 3

6

```
Remove["Global`*"]
```

```
S = {{-2, 2 / Sqrt[3]}, {2 / Sqrt[3], 2}}; S // MatrixForm
```

$$\begin{pmatrix} -2 & \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} & 2 \end{pmatrix}$$

```
X = {{x1}, {x2}}
```

```
{{x1}, {x2}}
```

```
Flatten[Transpose[X].S.X // Simplify][[1]]
```

$$-2\,x1^2 + \frac{4\,x1\,x2}{\sqrt{3}} + 2\,x2^2$$

```
syst = Eigensystem[S] // Simplify
```

$$\left\{\left\{-\frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right\}, \left\{\left\{-2-\sqrt{3}, 1\right\}, \left\{2-\sqrt{3}, 1\right\}\right\}\right\}$$

```
Dl = {{syst[[1]][[1]], 0}, {0, syst[[1]][[2]]}};  Dl // MatrixForm
```

$$\begin{pmatrix} -\frac{4}{\sqrt{3}} & 0 \\ 0 & \frac{4}{\sqrt{3}} \end{pmatrix}$$

```
n[v_] := v / Norm[v]
```

```
M = Transpose[{n[syst[[2]][[1]]], n[syst[[2]][[2]]] }];  M // MatrixForm
```

$$\begin{pmatrix} \frac{-2-\sqrt{3}}{\sqrt{1+\left(2+\sqrt{3}\right)^2}} & \frac{2-\sqrt{3}}{\sqrt{1+\left(2-\sqrt{3}\right)^2}} \\ \frac{1}{\sqrt{1+\left(2+\sqrt{3}\right)^2}} & \frac{1}{\sqrt{1+\left(2-\sqrt{3}\right)^2}} \end{pmatrix}$$

```
 M // N // MatrixForm
```

$$\begin{pmatrix} -0.965926 & 0.258819 \\ 0.258819 & 0.965926 \end{pmatrix}$$

```
N[Inverse[M]] == N[Transpose[M]]
```

```
True
```

```
Det[M] // N
```

```
-1.
```

```
Y = Inverse[M].X // Simplify
```

$$\left\{\left\{-\frac{1}{2}\sqrt{2+\sqrt{3}}\ \left(x1+\left(-2+\sqrt{3}\right)\ x2\right)\right\},\ \left\{\frac{1}{2}\sqrt{2-\sqrt{3}}\ \left(x1+\left(2+\sqrt{3}\right)\ x2\right)\right\}\right\}$$

```
Flatten[Transpose[X].S.X // Simplify][[1]] ==
 Flatten[Transpose[Y].Dl.Y // Simplify][[1]]
```

```
True
```

```
Y1 = {{y1}, {y2}}
```

```
{{y1}, {y2}}
```

```
Flatten[ Transpose[Y1].Dl.Y1 ] [[1]]
```

$$-\frac{4\ y1^2}{\sqrt{3}}+\frac{4\ y2^2}{\sqrt{3}}$$

```
N[%] == 52
```

$$-2.3094\ y1^2+2.3094\ y2^2==52$$

Hyperbel