Lösungen

1

a

```
f[x_]:=(t+1)x^101+t x^22+(t-1)x^5+t(x-1);
f[x]
```

$$t\,(-1+x)+(-1+t)\,x^5+t\,x^{22}+(1+t)\,x^{101}$$

```
f'[x]
```

$$t+5\,(-1+t)\,x^4+22\,t\,x^{21}+101\,(1+t)\,x^{100}$$

b

```
f'[x]/. {x->1,t->1}
```

$$225$$

c

```
f[x_]:=(a-1)x^(-a)+a x^a;
f[x]
```

$$(-1+a)\,x^{-a}+a\,x^a$$

```
f'[x]
```

$$-(-1+a)\,a\,x^{-1-a}+a^2\,x^{-1+a}$$

```
f'[x]/. {x->a}
```

$$-(-1+a)\,a^{-a}+a^{1+a}$$

d

```
f[x_]:=Sin[x] Log[x];
f[x]
```

$$\text{Log}[x]\,\text{Sin}[x]$$

```
f'[x]
```

$$\text{Cos}[x]\,\text{Log}[x]+\frac{\text{Sin}[x]}{x}$$

e

```
f'[x]/. x->Pi/4
```

$$\frac{2\sqrt{2}}{\pi} + \frac{\text{Log}\left[\frac{\pi}{4}\right]}{\sqrt{2}}$$

```
N[%]
```

```
0.729504
```

f

```
f[x_]:=Sin[x]/x^3+E^(Sin[x]);
f[x]
```

$$e^{\text{Sin}[x]} + \frac{\text{Sin}[x]}{x^3}$$

```
f'[x]
```

$$e^{\text{Sin}[x]}\,\text{Cos}[x] + \frac{\text{Cos}[x]}{x^3} - \frac{3\,\text{Sin}[x]}{x^4}$$

g

```
f'[x]/. x->Pi
```

$$-1 - \frac{1}{\pi^3}$$

```
N[%]
```

```
-1.03225
```

h

```
f[x_]:=E^(3 x^3-2x+1);
f[x]
```

$$e^{1-2x+3x^3}$$

```
f'[x]
```

$$e^{1-2x+3x^3}\,\left(-2+9x^2\right)$$

```
u = f'[x]/. x->1
```

$$7\,e^2$$

```
ArcTan[u]//N
```

```
1.55147
```

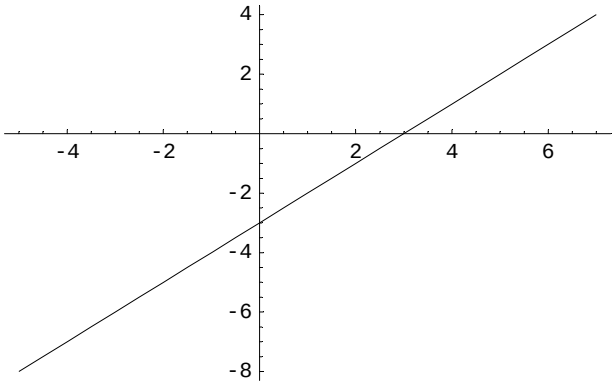
```
ArcTan[u]/Degree//N
```

```
88.8924
```

2

a

```
f[x_]:= (x-3) (x-1)/(x-1);
Plot[f[x],{x,-5,7}];
```




```
D[f[x],x]//Together
```

```
1
```

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

```
{}
```

```
N[%]
```

```
{}
```

```
D[f[x],{x,2}]//Together
```

```
0
```

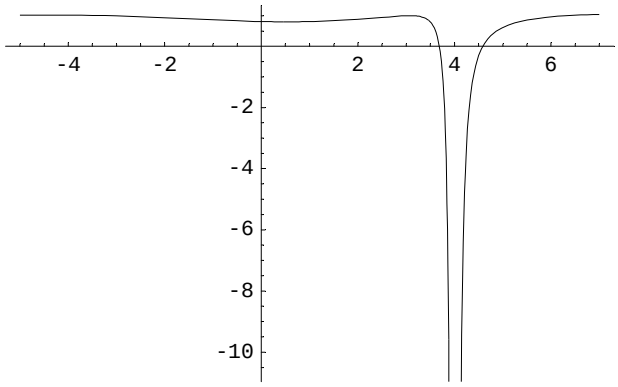
```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

```
{{}}
```

Keine Extreme uns WP

b

```
f[x_]:=(Sin[x](x-3)/(x (x-4)^2)+1);
Plot[f[x],{x,-5,7}];
```




```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

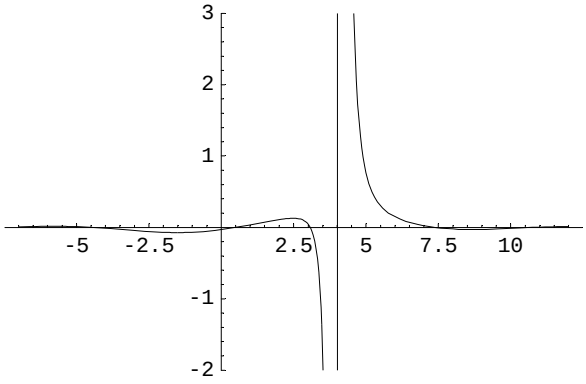
```
Solve::tdep : The equations appear to involve the
   variables to be solved for in an essentially non-algebraic way. Mehr…
```

$$\text{Solve}\left[\frac{(-3+x)\,\text{Cos}[x]}{(-4+x)^2\,x}-\frac{(-3+x)\,\text{Sin}[x]}{(-4+x)^2\,x^2}+\frac{\text{Sin}[x]}{(-4+x)^2\,x}-\frac{2\,(-3+x)\,\text{Sin}[x]}{(-4+x)^3\,x}==0,\{x\}\right]$$

```
Numerator[D[f[x],x]]//Together
```

$$\frac{12\,x\,\text{Cos}[x]-7\,x^2\,\text{Cos}[x]+x^3\,\text{Cos}[x]-12\,\text{Sin}[x]+9\,x\,\text{Sin}[x]-2\,x^2\,\text{Sin}[x]}{(-4+x)^3\,x^2}$$

```
Plot[Evaluate[Numerator[D[f[x],x]]],{x,-7,12},PlotRange->{-2,3}];
```




```
FindRoot[Evaluate[D[f[x],x]==0],{x,3}]
```

$\{x \to 3.07544\}$

```
FindRoot[Evaluate[D[f[x],x]==0],{x,8}]
```

$\{x \to 7.39263\}$

```
FindRoot[Evaluate[D[f[x],x]==0],{x,1}]
```

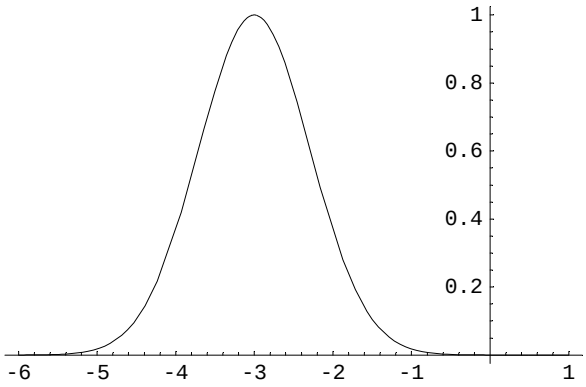
$\{x \to 0.505511\}$

Nur 1. pos Extremum 0.5055...

c

```
f[x_]:=E^(-(x+3)^2);
```

```
Plot[E^(-(x+3)^2),{x,-6,1}];
```




```
D[f[x],x]//Together
```

$$-2\, e^{-(3+x)^2}\, (3+x)$$

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

$$\{\{x \to -3\}\}$$

```
N[%]
```

$$\{\{x \to -3.\}\}$$

```
D[f[x],{x,2}]//Together
```

$$2\, e^{-(3+x)^2}\, (17 + 12\, x + 2\, x^2)$$

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

$$\left\{\left\{x \to \frac{1}{2}\left(-6-\sqrt{2}\right)\right\}, \left\{x \to \frac{1}{2}\left(-6+\sqrt{2}\right)\right\}\right\}$$

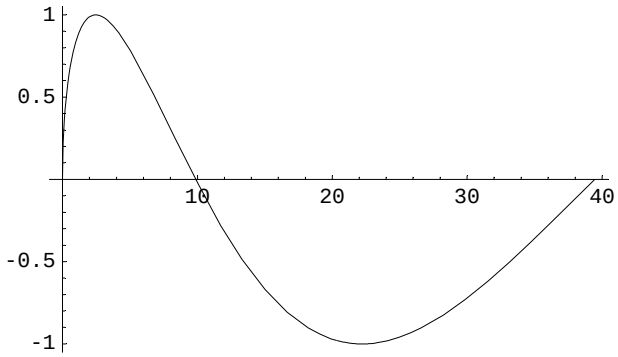
```
N[%]
```

$$\{\{x \to -3.70711\}, \{x \to -2.29289\}\}$$

d

```
f[x_]:=Sin[Sqrt[x]];
```

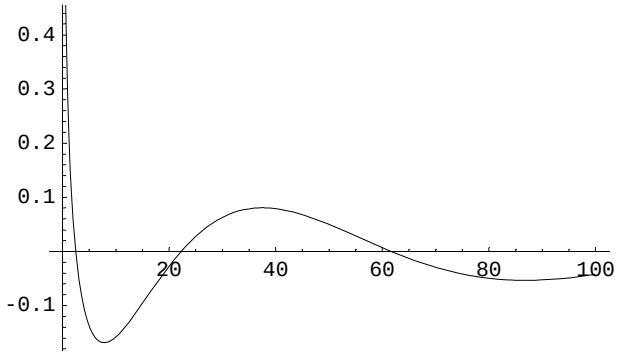
Plot[Sin[Sqrt[x]],{x,0,4Pi^2}];



D[f[x],x]//Together

$$\frac{\text{Cos}\left[\sqrt{x}\right]}{2\sqrt{x}}$$

Plot[Evaluate[D[f[x],x]],{x,0,100}];



Solve[Evaluate[D[f[x],x]==0],{x}]

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

$$\left\{\left\{x \to \frac{\pi^2}{4}\right\}\right\}$$

N[%]

$$\{\{x \to 2.4674\}\}$$

D[f[x],{x,2}]//Together

$$\frac{-\text{Cos}\left[\sqrt{x}\right] - \sqrt{x}\ \text{Sin}\left[\sqrt{x}\right]}{4\,x^{3/2}}$$

FindRoot[Evaluate[D[f[x],{x,1}]==0],{x,20}]

$$\{x \to 22.2066\}$$

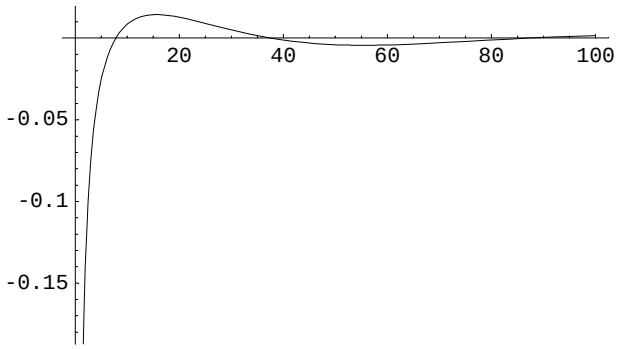
FindRoot[Evaluate[D[f[x],{x,1}]==0],{x,50}]

$$\{x \to 61.685\}$$

```
FindRoot[Evaluate[D[f[x],{x,1}]==0],{x,100}]
```

$\{x \to 120.903\}$

```
Plot[Evaluate[D[f[x],{x,2}]],{x,0,100}];
```




```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,5}]
```

$\{x \to 7.83096\}$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,40}]
```

$\{x \to 37.4697\}$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,90}]
```

$\{x \to 86.8226\}$

3

a

```
f[x_]:=2 (x-3) (x-2)/(2x-4)^2;
{Limit[f[x],{x->0}],Limit[f[x],{x->Infinity}]}
```

$\left\{\left\{\frac{3}{4}\right\}, \left\{\frac{1}{2}\right\}\right\}$

b

```
f[x_]:=Sin[x] (x-3) /(x (x-4)^2) +1;
{Limit[f[x],{x->0}],Limit[f[x],{x->Infinity}]}
```

$\left\{\left\{\frac{13}{16}\right\}, \{1\}\right\}$

c

```
f[x_]:=E^(-x^2+2 x) x+(Cos[x]+1)/x;
{Limit[f[x],{x->0}],Limit[f[x],{x->Infinity}]}
```

$$\{\{\infty\}, \{0\}\}$$

d

```
f[x_]:=Sin[E^(-x)+1/x];
{Limit[f[x],{x->0}],Limit[f[x],{x->Infinity}]}
```

$$\{\{\text{Interval}[\{-\cos[1]-\sin[1], \cos[1]+\sin[1]\}]\}, \{0\}\}$$

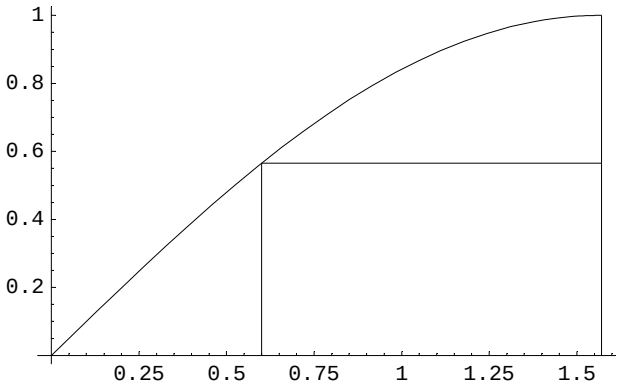
4

a

I

```
Remove["Global`*"]
```

```
Plot[Sin[x],{x,0,Pi/2},Epilog->{Line[{{Pi/2,0},{Pi/2,1}}],Line[{{0.6,0},{0.6,Sin[0.
6]}}],Line[{{0.6,Sin[0.6]},{Pi/2,Sin[0.6]}}]}];
```



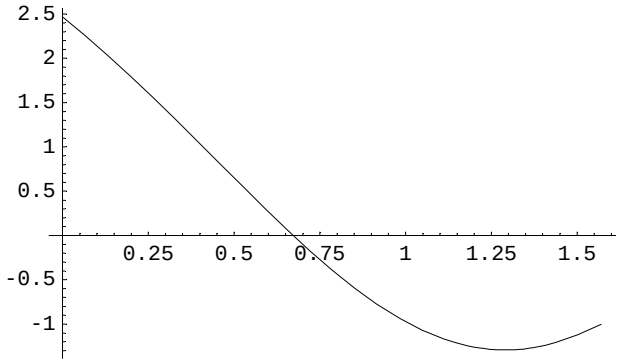

```
V[x_]:=(Pi/2-x) Sin[x] (Pi/2-x+Sin[x]); V[x]
```

$$\left(\frac{\pi}{2}-x\right)\text{Sin}[x]\left(\frac{\pi}{2}-x+\text{Sin}[x]\right)$$

```
D[V[x],x]//Simplify
```

$$-\text{Sin}[x]\,(\pi-2x+\text{Sin}[x])+\frac{1}{4}\,(\pi-2x)\,\text{Cos}[x]\,(\pi-2x+4\,\text{Sin}[x])$$

```
Plot[Evaluate[D[V[x],x]],{x,0,Pi/2}];
```




```
FindRoot[Evaluate[D[V[x],x]==0],{x,0.7}]
```

$\{x \to 0.672361\}$

II

```
Remove["Global`*"]
```

```
f[x_]:=Sin[x]; g[x_,x0_]:= Evaluate[f[x0]+(D[f[u],u]/.u->x0)(x-x0)]; g[x,x0]
```

$(x - x0)\ \text{Cos}[x0] + \text{Sin}[x0]$

```
g[u, v]
```

$(u - v)\ \text{Cos}[v] + \text{Sin}[v]$

```
PyO[x0_]:=g[Pi/2,x0];
PyO[x0]
```

$\left(\frac{\pi}{2} - x0\right)\ \text{Cos}[x0] + \text{Sin}[x0]$

```
PyO[2]
```

$\left(-2 + \frac{\pi}{2}\right)\ \text{Cos}[2] + \text{Sin}[2]$

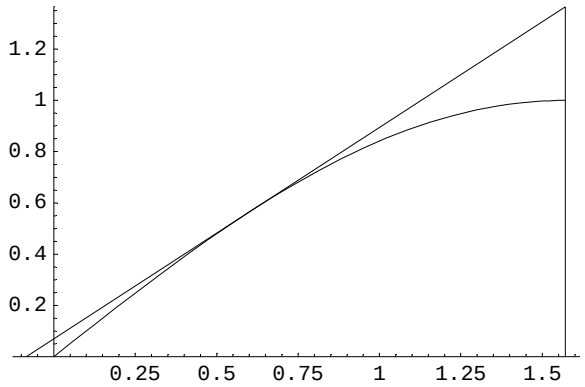
```
xU=x/.(Solve[g[x,x0]==0,{x}]//Flatten)
```

$\text{Sec}[x0]\ (x0\ \text{Cos}[x0] - \text{Sin}[x0])$

```
x1U[x0_] := x /. (Solve[g[x, x0] == 0, {x}] // Flatten); x1U[z]
```

$\text{Sec}[z]\ (z\ \text{Cos}[z] - \text{Sin}[z])$

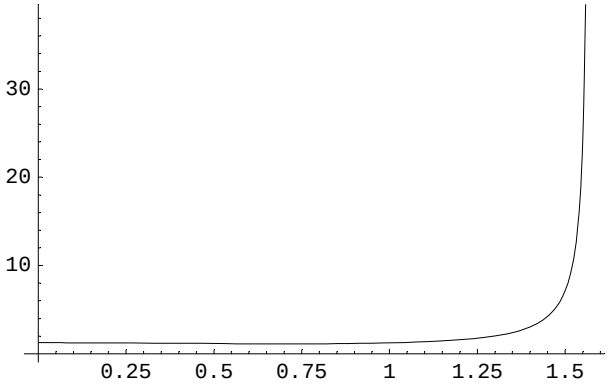
```
x0=0.6;
Plot[Sin[x],{x,xU,Pi/2},Epilog->{Line[{{xU,0},{Pi/2,PyO[x0]}}],
Line[{{Pi/2,0},{Pi/2,PyO[x0]}}]},PlotRange->{0,PyO[x0]}];
```



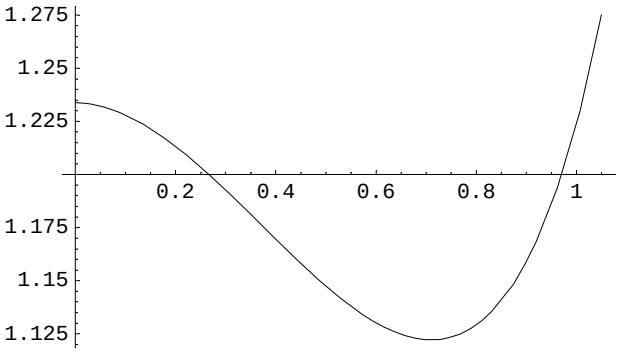

```
A[x1_]:=PyO[x1] (Pi/2- x1U[x1])/2; A[x2]
```

$$\frac{1}{2}\left(\frac{\pi}{2}-\text{Sec}[\text{x2}]\,(\text{x2}\,\text{Cos}[\text{x2}]-\text{Sin}[\text{x2}])\right)\left(\left(\frac{\pi}{2}-\text{x2}\right)\text{Cos}[\text{x2}]+\text{Sin}[\text{x2}]\right)$$

```
Plot[Evaluate[A[u]], {u, 0, Pi / 2}];
```




```
Plot[Evaluate[A[u]], {u, 0, 1.05}];
```




```
D[Evaluate[A[u]], {u}]
```

$$-\frac{1}{2}\left(\frac{\pi}{2}-\text{u}\right)\left(\frac{\pi}{2}-\text{Sec}[\text{u}]\,(\text{u}\,\text{Cos}[\text{u}]-\text{Sin}[\text{u}])\right)\text{Sin}[\text{u}]+$$
$$\frac{1}{2}\left(\left(\frac{\pi}{2}-\text{u}\right)\text{Cos}[\text{u}]+\text{Sin}[\text{u}]\right)(\text{u}\,\text{Tan}[\text{u}]-\text{Sec}[\text{u}]\,(\text{u}\,\text{Cos}[\text{u}]-\text{Sin}[\text{u}])\,\text{Tan}[\text{u}])$$

```
FindRoot[D[Evaluate[A[u]], {u}], {u, 0.6}]
```

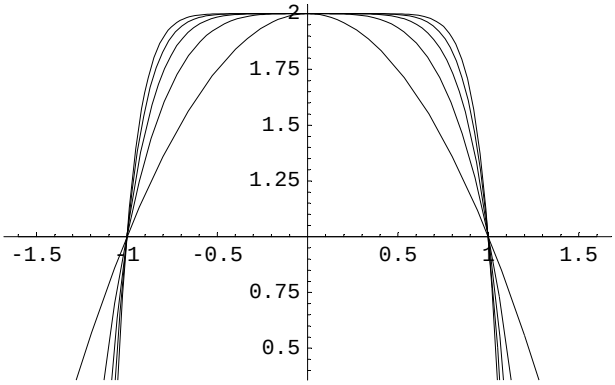
$$\{u \to 0.710463\}$$

b

```
f[x_,n_]:= -x^(2 n)+2; f[x,n]
```

$$2 - x^{2\,n}$$

```
Plot[Evaluate[Table[f[x, n], {n, 0, 5}]], {x, -1.6, 1.6}];
```




```
2 (x + f[x, n]) // Expand
```

$$4 + 2\,x - 2\,x^{2\,n}$$

```
rechteck[x_, n_] := 2 (x + f[x, n])
rechteck[x, n] // Expand
```

$$4 + 2\,x - 2\,x^{2\,n}$$

```
D[rechteck[x, n], {x}] // Expand
```

$$2 - 4\,n\,x^{-1+2\,n}$$

```
Evaluate[Table[rechteck[x, n], {n, 0, 5}]]
```

$$\{2\,(1+x),\ 2\,(2+x-x^2),\ 2\,(2+x-x^4),\ 2\,(2+x-x^6),\ 2\,(2+x-x^8),\ 2\,(2+x-x^{10})\}$$

```
Evaluate[Table[D[rechteck[x, n], {x}] // Expand, {n, 0, 5}]]
```

$$\{2,\ 2-4\,x,\ 2-8\,x^3,\ 2-12\,x^5,\ 2-16\,x^7,\ 2-20\,x^9\}$$

```
Evaluate[Table[Expand[D[rechteck[x, n], {x}]] == 0, {n, 0, 5}]]
```

$$\{\text{False},\ 2-4\,x == 0,\ 2-8\,x^3 == 0,\ 2-12\,x^5 == 0,\ 2-16\,x^7 == 0,\ 2-20\,x^9 == 0\}$$

```
Evaluate[Table[Solve[Expand[D[rechteck[x, n], {x}]] == 0, {x}], {n, 0, 5}]]
```

$$\{\{\}, \{\{x \to \frac{1}{2}\}\}, \{\{x \to \left(-\frac{1}{2}\right)^{2/3}\}, \{x \to \frac{1}{2^{2/3}}\}, \{x \to -\frac{(-1)^{1/3}}{2^{2/3}}\}\},$$
$$\{\{x \to -\left(-\frac{1}{6}\right)^{1/5}\}, \{x \to \frac{1}{6^{1/5}}\}, \{x \to \frac{(-1)^{2/5}}{6^{1/5}}\}, \{x \to -\frac{(-1)^{3/5}}{6^{1/5}}\}, \{x \to \frac{(-1)^{4/5}}{6^{1/5}}\}\},$$
$$\{\{x \to -\left(-\frac{1}{2}\right)^{3/7}\}, \{x \to \frac{1}{2^{3/7}}\}, \{x \to -\frac{(-1)^{1/7}}{2^{3/7}}\},$$
$$\{x \to \frac{(-1)^{2/7}}{2^{3/7}}\}, \{x \to \frac{(-1)^{4/7}}{2^{3/7}}\}, \{x \to -\frac{(-1)^{5/7}}{2^{3/7}}\}, \{x \to \frac{(-1)^{6/7}}{2^{3/7}}\}\},$$
$$\{\{x \to -\left(-\frac{1}{10}\right)^{1/9}\}, \{x \to \frac{1}{10^{1/9}}\}, \{x \to \frac{(-1)^{2/9}}{10^{1/9}}\}, \{x \to -\frac{(-1)^{1/3}}{10^{1/9}}\}, \{x \to \frac{(-1)^{4/9}}{10^{1/9}}\},$$
$$\{x \to -\frac{(-1)^{5/9}}{10^{1/9}}\}, \{x \to \frac{(-1)^{2/3}}{10^{1/9}}\}, \{x \to -\frac{(-1)^{7/9}}{10^{1/9}}\}, \{x \to \frac{(-1)^{8/9}}{10^{1/9}}\}\}\}$$

```
Evaluate[Table[Solve[Expand[D[rechteck[x, n], {x}]] == 0, {x}], {n, 0, 5}]] // N //
 MatrixForm
```

$$\begin{pmatrix} \{\} \\ \{\{x \to 0.5\}\} \\ \{\{x \to -0.31498 + 0.545562\,i\}, \{x \to 0.629961\}, \{x \to -0.31498 - 0.545562\,i\}\} \\ \{\{x \to -0.565363 - 0.41076\,i\}, \{x \to 0.698827\}, \{x \to 0.215949 + 0.664624\,i\}, \{x \to 0.215949 - \\ \{\{x \to -0.165332 - 0.724369\,i\}, \{x \to 0.742997\}, \{x \to -0.669417 - 0.322374\,i\}, \{x \to 0.463251 \\ \{\{x \to -0.72757 - 0.264814\,i\}, \{x \to 0.774264\}, \{x \to 0.59312 + 0.497687\,i\}, \{x \to -0.387132 - \end{pmatrix}$$

```
tab1 = ToRadicals[
  Flatten[Evaluate[Table[Solve[Expand[D[rechteck[x, n], {x}]] == 0, {x}], {n, 0, 5}]]]]
```

$$\{x \to \frac{1}{2}, x \to \left(-\frac{1}{2}\right)^{2/3}, x \to \frac{1}{2^{2/3}}, x \to -\frac{(-1)^{1/3}}{2^{2/3}}, x \to -\left(-\frac{1}{6}\right)^{1/5}, x \to \frac{1}{6^{1/5}}, x \to \frac{(-1)^{2/5}}{6^{1/5}},$$
$$x \to -\frac{(-1)^{3/5}}{6^{1/5}}, x \to \frac{(-1)^{4/5}}{6^{1/5}}, x \to -\left(-\frac{1}{2}\right)^{3/7}, x \to \frac{1}{2^{3/7}}, x \to -\frac{(-1)^{1/7}}{2^{3/7}}, x \to \frac{(-1)^{2/7}}{2^{3/7}},$$
$$x \to \frac{(-1)^{4/7}}{2^{3/7}}, x \to -\frac{(-1)^{5/7}}{2^{3/7}}, x \to \frac{(-1)^{6/7}}{2^{3/7}}, x \to -\left(-\frac{1}{10}\right)^{1/9}, x \to \frac{1}{10^{1/9}}, x \to \frac{(-1)^{2/9}}{10^{1/9}},$$
$$x \to -\frac{(-1)^{1/3}}{10^{1/9}}, x \to \frac{(-1)^{4/9}}{10^{1/9}}, x \to -\frac{(-1)^{5/9}}{10^{1/9}}, x \to \frac{(-1)^{2/3}}{10^{1/9}}, x \to -\frac{(-1)^{7/9}}{10^{1/9}}, x \to \frac{(-1)^{8/9}}{10^{1/9}}\}$$

```
tab2 = Table[
  ToRadicals[Flatten[Evaluate[Table[Solve[Expand[D[rechteck[x, n], {x}]] == 0, {x}],
       {n, 0, 5}]]]][[n]][[2]], {n, 1, Length[tab1]}]
```

$$\{\frac{1}{2}, \left(-\frac{1}{2}\right)^{2/3}, \frac{1}{2^{2/3}}, -\frac{(-1)^{1/3}}{2^{2/3}}, -\left(-\frac{1}{6}\right)^{1/5}, \frac{1}{6^{1/5}}, \frac{(-1)^{2/5}}{6^{1/5}}, -\frac{(-1)^{3/5}}{6^{1/5}}, \frac{(-1)^{4/5}}{6^{1/5}},$$
$$-\left(-\frac{1}{2}\right)^{3/7}, \frac{1}{2^{3/7}}, -\frac{(-1)^{1/7}}{2^{3/7}}, \frac{(-1)^{2/7}}{2^{3/7}}, \frac{(-1)^{4/7}}{2^{3/7}}, -\frac{(-1)^{5/7}}{2^{3/7}}, \frac{(-1)^{6/7}}{2^{3/7}}, -\left(-\frac{1}{10}\right)^{1/9},$$
$$\frac{1}{10^{1/9}}, \frac{(-1)^{2/9}}{10^{1/9}}, -\frac{(-1)^{1/3}}{10^{1/9}}, \frac{(-1)^{4/9}}{10^{1/9}}, -\frac{(-1)^{5/9}}{10^{1/9}}, \frac{(-1)^{2/3}}{10^{1/9}}, -\frac{(-1)^{7/9}}{10^{1/9}}, \frac{(-1)^{8/9}}{10^{1/9}}\}$$

```
tab2[[5]]
```

$$-\left(-\frac{1}{6}\right)^{1/5}$$

```
If[Element[tab2[[1]], Reals], tab2[[1]], {}]
```

$$\frac{1}{2}$$

```
Table[If[Element[tab2[[n]], Reals], tab2[[n]], {}], {n, 1, Length [tab2]}] // Flatten
```

$$\left\{\frac{1}{2}, \frac{1}{2^{2/3}}, \frac{1}{6^{1/5}}, \frac{1}{2^{3/7}}, \frac{1}{10^{1/9}}\right\}$$

```
Table[If[Element[tab2[[n]], Reals], tab2[[n]], {}], {n, 1, Length [tab2]}] // Flatten //
 N
```

```
{0.5, 0.629961, 0.698827, 0.742997, 0.774264}
```